Large Component QCD and Theoretical Framework of Heavy Quark Effective Field Theory

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Abstract

Based on a large component QCD derived directly from full QCD by integrating over the small components of quark fields with $|\mathbf{p}| < E + m_Q$, an alternative quantization procedure is adopted to establish a basic theoretical framework of heavy quark effective field theory (HQEFT) in the sense of effective quantum field theory. The procedure concerns quantum generators of Poincare group, Hilbert and Fock space, anticommutations and velocity super-selection rule, propagator and Feynman rules, finite mass corrections, trivialization of gluon couplings and renormalization of Wilson loop. The Lorentz invariance and discrete symmetries in HQEFT are explicitly illustrated. Some new symmetries in the infinite mass limit are discussed. Weak transition matrix elements and masses of hadrons in HQEFT are well defined to display a manifest spin-flavor symmetry and $1/m_Q$ corrections. A simple trace formulation approach is explicitly demonstrated by using LSZ reduction formula in HQEFT, and shown to be very useful for parameterizing the transition form factors via $1/m_Q$ expansion. As the heavy quark and antiquark fields in HQEFT are treated on the same footing in a fully symmetric way, the quark-antiquark coupling terms naturally appear and play important roles for simplifying the structure of transition matrix elements, and for understanding the introduction of 'dressed heavy quark' - hadron duality. In the case that the 'longitudinal' and 'transverse' residual momenta of heavy quark are at the same order of power counting, HQEFT provides a consistent approach for systematically analyzing heavy quark expansion in terms of $1/m_Q$. Some interesting features in applications of HQEFT to heavy hadron systems are briefly outlined.

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I. INTRODUCTION

Great experimental and theoretical progresses have been achieved in heavy quark physics[1, 2]. As more and more accurate experimental data become available in the future based on B-factories and other colliders, it requires more precise theoretical calculations for hadronic matrix elements since physical observables can only be measured from hadronic physics due to quark confinement. Thus one of the important issues concerns the treatment of nonperturbative QCD as heavy hadrons are bound states of quarks with a typical binding energy $\bar{\Lambda} \sim 2\Lambda_{QCD} \simeq 600$ MeV. The high energy behavior of QCD is well understood from the asymptotic freedom of QCD. To understand low energy dynamics of QCD, one needs to develop an appropriate method for treating non-perturbative QCD effects at low energies. As the relative momentum of heavy quark within the hadrons is at the order of binding energy scale which is smaller than the heavy quark mass. For heavy bottom and charm quarks, their masses $m_Q = m_b,\ m_c$ are much larger than the QCD energy scale $m_Q \gg \bar{\Lambda} \sim 2\Lambda_{QCD}$. In order to study low energy behavior of heavy quark, it is useful to express the four momentum of heavy quark as $p = m_Q v + k$ with $v^2 = 1$. Here k denotes the residual momentum or relative momentum when ν_μ is taken to be the velocity of heavy hadron at the rest frame $v = (1, 0, 0, 0)$. As a heavy quark within the hadron has a typical residual momentum $|\mathbf{k}| \sim \bar{\Lambda} \ll m_Q$, which motivates us to develop an effective field theory to treat the non-perturbative QCD effects for heavy quarks. A large component QCD (LCQCD) treating large component effective heavy quark and antiquark fields on the same footing can be derived directly from full QCD at $|\mathbf{p}| \ll E + m_Q$ or $|\mathbf{k}| \ll 2m_Q + k^0$. For the case that the heavy quark is nearly on-mass shell, i.e., $k^0 \sim |\mathbf{k}|^2/2m_Q$, LCQCD is shown to behave like a non-relativistic QCD (NRQCD). When the heavy quark goes to off-mass shell with $k^0 \gg |\mathbf{k}|^2/2m_Q$ (i.e., $|\mathbf{k}| \ll \sqrt{2k^0 m_Q} \sim \sqrt{2\bar{\Lambda} m_Q}$ for $k^0 \sim \bar{\Lambda}$), the typical case occurs in the heavy-light hadron system in which the "longitudinal" and "transverse" residual momenta of heavy quark are at the same order of power counting (i.e., $k^0 \sim |\mathbf{k}| \sim \bar{\Lambda} \ll m_Q$), then LCQCD can be treated as a heavy quark effective field theory (HQEFT)[3] via the heavy quark expansion in terms of $1/m_Q$,

which has been applied to deal with heavy-light hadron systems[4, 5, 6, 7, 8, 9]. The leading term of effective lagrangian in HQEFT characterizes the behavior of heavy quarks in the infinite mass limit[10, 11] and coincides with the standard one[12] which explicitly displays the heavy quark spin-flavor symmetry[13, 14, 15].

The effective Lagrangian in HQEFT[3] differs from the widely used one in the literatures [16] which was shown to be constructed based on the assumption that particle and antiparticle numbers are separately conserved [17]. Thus the differences between two effective theories arise mainly from the quark-antiquark mixed interaction terms. Therefore, it is of interest to show explicitly their differences and relations. Namely, in which case the effective Lagrangian in HQEFT can recover the widely used heavy quark effective Lagrangian. Theoretically, it is known from quantum field theories that only in the infinity mass limit the quark and antiquark numbers become separately conserved as the quark and antiquark interactions approach to decouple. In other words, with finite quark mass in the real world, it requires one to keep the quark-antiquark interaction terms and to treat the quark and antiquark fields on the same footing in the sense of effective quantum field theory at low energies. On the other hand, the effects of quark-antiquark interaction terms at low energies should also distinguish from the Wilson coefficient functions which mainly reflect the perturbative QCD corrections at relatively high energy scales.

As LCQCD is directly derived from QCD, all contributions of the field components, large and small, 'particle' (positive energy part) and 'antiparticle' (negative energy part), have carefully been dealt with in the effective Lagrangian. Especially, the effective heavy quark and antiquark fields are treated on the same footing in a fully symmetric way, the resulting effective Lagrangian shall provide a complete description for both quark and antiquark fields. In fact, HQEFT based on LCQCD has lead to a number of successful phenomenological applications, such as: a reliable extraction for the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$[4, 5, 6], a consistent explanation for the lifetime differences among bottom hadrons[7], a good approximated scaling law for the decay constants of heavy mesons[8], an interesting observation for useful relations of form factors in heavy to light hadron transitions[9]. It then strongly motivates us to present a complete theoretical framework of HQEFT in

the sense of effective quantum field theory at low energies when the "longitudinal" and "transverse" residual momenta of heavy quark are at the same order of power counting in the $1/m_Q$ expansion, and to demonstrate a systematic approach for evaluating the symmetry breaking effects due to $1/m_Q$ corrections. This comes to our main purpose in the present paper.

The paper is organized as follows: in section 2, we present a complete derivation for the effective Lagrangian of LCQCD directly from full QCD by integrating over the small components of quark and antiquark fields, it is explicitly shown how the large component quark-antiquark coupling terms are naturally resulted in the effective Lagrangian of LCQCD. The Lorentz invariance of LCQCD is explicitly illustrated, which leads to a super-equivalence of LCQCD with different velocities. We then discuss the special cases in which the quark components or antiquark components decouple from effective Lagrangian, so that the effective Lagrangian is reduced to the widely used one in the literatures. Our special attention is paid to consider two interesting cases which are corresponding to two energy regions of heavy quark. We will show how the two energy regions make LCQCD to be dealt with as two effective theories: one like a non-relativistic QCD (NRQCD), and another as a heavy quark effective field theory (HQEFT). A detailed discussion is given for HQEFT with large component quark-antiquark coupling terms. Of particular, we explicitly demonstrate the important effects of quark-antiquark coupling terms in HQEFT when the residual energy and momentum are at the same order of power counting, i.e., $k^0 \sim |\mathbf{k}| \sim \bar{\Lambda} \ll m_Q$. The quantization of HQEFT is carried out in section 3, as the large component effective heavy quark and antiquark fields $Q_v(x)$ and $\bar{Q}_v(x)$ concern the additional arbitrary unit time-like vector v_μ, the canonical quantization is not quite clear for it, we then adopt an alternative quantization procedure, which concerns quantum generators of Poincare group, Hilbert and Fock space, anticommutations and velocity superselection rule, propagator and discrete symmetries in HQEFT. In section 4, a basic theoretical framework of HQEFT is established based on the quantization of HQEFT and Feynman rules as well as the finite mass corrections in the heavy quark expansion of $1/m_Q$. The trivialization of gluon couplings and renormalization of Wilson loop are explicitly demonstrated.

Especially, the current operators via $1/m_Q$ expansion in HQEFT are discussed in detail. It is emphasized that in the computation of the coefficient functions at subleading order in $1/m_Q$, the large component quark-antiquark mixing terms in the current operators must be kept as they will receive contributions from the terms of the quark-antiquark coupling terms in the effective Lagrangian via virtual quark or antiquark field exchanges. It is also shown that there actually exist more symmetries in HQEFT at infinite mass limit, in addition to the well-known spin-flavor and angular momentum symmetries. It section 5, the weak transition matrix elements in HQEFT are well defined to display the manifest spin-flavor symmetry and to exhibit a systematic heavy quark expansion in terms of $1/m_Q$. All relevant operators up to order of $1/m_Q^2$ are presented explicitly. An alternative definition for the mass of heavy hadron is provided in HQEFT via $1/m_Q$ expansion. Some important transition matrix elements at zero recoil in HQEFT are investigated, it is shown that $1/m_Q$ order corrections are automatically absent at zero recoil point in HQEFT even without analyzing the concrete Lorentz structure and the form factors of the transition matrix elements. Of particualr, a simple trace formulation for evaluating the transition matrix elements is explicitly demonstrated by using the LSZ reduction formula for heavy hadron states in HQEFT. As a consequence, the universal Isgur-Wise function is shown to relate explicitly the overlapping integral between wave functions of initial and final state mesons. We then show that the simple trace formulation approach becomes very powerful for parameterizing the transition form factors via $1/m_Q$ expansion in HQEFT. Especially, as the effective quark and antiquark fields are treated symmetrically on the same footing, it simplifies the structure of transition matrix elements in the heavy quark expansion in terms of $1/m_Q$. This is because the operators appearing in the effective Lagrangian and effective current contain only terms with even powers of $\not{D}_\perp$ when the contributions of quark-antiquark interaction terms in HQEFT are considered. As HQEFT can be applied to treat off-mass shell heavy quarks, the concept of "dressed heavy quark" with well defined dressed quark mass $\hat{m}_Q = m_Q + \bar{\Lambda} = \lim_{m_Q \to \infty} m_H$ is introduced to characterize the confining binding effects of non-perturbative QCD

at low energies, here $m_H = \hat{m}_Q\left(1 + O(1/m_Q^2)\right)$ is the heavy hadron mass. The 'dressed heavy quark'-hadron duality is found to be more reasonable than the naive quark-hadron duality. Consequently, HQEFT allows us to make a systematic and consistent evaluation for the symmetry breaking corrections caused by the finite mass of heavy quark. Some interesting features and phenomena for the application of HQEFT are briefly outlined. Our conclusions and remarks are presented in the final section.

II. LARGE COMPONENT QCD

A. Effective Lagrangian of LCQCD from full QCD

For completeness, we follow ref.[3] to derive in a more systematic way the effective lagrangian of LCQCD from full QCD. The Lagrangian in full QCD is

$$\mathcal{L}_{QCD} = \bar{Q}(i\not{D} - m_Q)Q + \mathcal{L}_{light} \tag{2.1}$$

where Q denotes the heavy quark field and $\mathcal{L}_{light}$ represents the remaining part. It is well known that the Dirac fermion fields contain particle and antiparticle components which correspond to positive and negative energy parts. Formally, one can always write the field Q into two parts

$$Q = Q^{(+)} + Q^{(-)}, \tag{2.2}$$

with $Q^{(+)}$ and $Q^{(-)}$ relating mainly to the positive energy part and negative energy part of classical solutions respectively. In the case of the free quark field, they are corresponding to two solutions of Dirac equation

$$(i\not{\partial} - m_Q)Q^{(\pm)} = 0. \tag{2.3}$$

Here $Q^{(+)}$ and $Q^{(-)}$ are known as "quark" and "antiquark" fields respectively with the following explicit solutions

$$Q^{(+)}(x) = \int \frac{d^3p}{(2\pi)^3}\frac{m}{p^0}\sum_s b_s(p)u_s(p)e^{-ip\cdot x}, \tag{2.4}$$

$$Q^{(-)}(x) = \int \frac{d^3p}{(2\pi)^3}\frac{m}{p^0}\sum_s d_s^\dagger(p)v_s(p)e^{ip\cdot x}, \tag{2.5}$$

where s is spin index, b_s and d_s are the annihilation and creation operators respectively. u_s and v_s are four-component spinors. In the Dirac representation, they are given by the following explicit forms

$$u_s(p) = \sqrt{\frac{E+m_Q}{2m_Q}} \begin{pmatrix} 1 \\ \frac{\sigma\cdot\mathbf{p}}{E+m_Q} \end{pmatrix} \varphi_s, \tag{2.6}$$

$$v_s(p) = \sqrt{\frac{E+m_Q}{2m_Q}} \begin{pmatrix} \frac{\sigma\cdot\mathbf{p}}{E+m_Q} \\ 1 \end{pmatrix} \chi_s \tag{2.7}$$

with φ_s being the two component Pauli spinor field that annihilates a heavy quark, and χ_s being the Pauli spinor field that creates a heavy antiquark.

Introducing the projection operators $P_\pm = (1 \pm \not{v})/2$ satisfying $P_\pm^2 = 1$ with v^μ being an arbitrary unit vector $v^2 = 1$, we can decompose the quark fields $Q^{(\pm)}$ into the following forms

$$\begin{aligned} Q^{(+)} &\equiv \big(\frac{1+\not{v}}{2} + \frac{1-\not{v}}{2}\big)Q^{(+)} = \hat{Q}_v^{(+)} + R_v^{(+)}, \\ Q^{(-)} &\equiv \big(\frac{1-\not{v}}{2} + \frac{1+\not{v}}{2}\big)Q^{(-)} = \hat{Q}_v^{(-)} + R_v^{(-)} \end{aligned} \tag{2.8}$$

with

$$\hat{Q}_v^{(\pm)} \equiv \frac{1\pm\not{v}}{2} Q^{(\pm)}, \qquad R_v^{(\pm)} \equiv \frac{1\mp\not{v}}{2} Q^{(\pm)} \tag{2.9}$$

Thus the initial quark field Q can be rewritten as

$$Q \equiv \hat{Q}_v + R_v \tag{2.10}$$

with

$$\hat{Q}_v = \hat{Q}_v^{(+)} + \hat{Q}_v^{(-)}, \qquad R_v = R_v^{(+)} + R_v^{(-)}. \tag{2.11}$$

For free quark case, taking $v = (1, 0, 0, 0)$, the projected quark fields get the following forms in the momentum space

$$\hat{Q}_v^{(+)} \to \frac{1+\not{v}}{2} u_s(p) = \sqrt{\frac{E+m_Q}{2m_Q}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi_s, \tag{2.12}$$

$$R_v^{(+)} \to \frac{1-\not{v}}{2} u_s(p) = \sqrt{\frac{E+m_Q}{2m_Q}} \begin{pmatrix} 0 \\ \frac{\sigma\cdot\mathbf{p}}{E+m_Q} \end{pmatrix} \varphi_s, \tag{2.13}$$

$$R_v^{(-)} \rightarrow \frac{1+\not v}{2} v_s(p) \; = \; \sqrt{\frac{E+m_Q}{2m_Q}} \begin{pmatrix} \frac{\sigma\cdot\mathbf{p}}{E+m_Q} \\ 0 \end{pmatrix} \chi_s, \tag{2.14}$$

$$\hat{Q}_v^{(-)} \rightarrow \frac{1-\not v}{2} v_s(p) \; = \; \sqrt{\frac{E+m_Q}{2m_Q}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \chi_s. \tag{2.15}$$

which shows that in the case $|\mathbf{p}| \ll E+m_Q$ the effective heavy fields $\hat{Q}_v^{(+)}$ and $\hat{Q}_v^{(-)}$ are the corresponding "large components" of "quark" and "antiquark" fields, while $R_v^{(+)}$ and $R_v^{(-)}$ are the corresponding "small components" .

It is also useful to decompose the derivative operator into two parts

$$\not D = \not D_{\parallel} + \not D_{\perp} \tag{2.16}$$

with

$$\not D_{\parallel} \equiv \not v v \cdot D, \quad \not D_{\perp} \equiv \not D - \not v(v\cdot D), \tag{2.17}$$

They satisfy the commutation relations

$$[\not v, \not D_{\parallel}] = 0, \qquad \{\not v, \not D_{\perp}\} = 0, \tag{2.18}$$

Using this property, we may relate the field components $R_v^{(\pm)}$ ($\bar{R}_v^{(\pm)}$) with $\hat{Q}_v^{(\pm)}$ ($\bar{\hat{Q}}_v^{(\pm)}$) via the following equations:

$$(i\not D_{\parallel} - m_Q) R_v^{(\pm)} + i\not D_{\perp} \hat{Q}_v^{(\pm)} = 0, \tag{2.19}$$

$$\bar{R}_v^{(\pm)}(-i\overleftarrow{\not D}_{\parallel} - m_Q) - \bar{\hat{Q}}_v^{(\pm)} i\overleftarrow{\not D}_{\perp} = 0. \tag{2.20}$$

which are actually the half part of Dirac equation of motion. With this relations, R_v becomes $1/m_Q$ suppressed relative to $\hat{Q}_v$. Thus $\hat{Q}_v^{(\pm)}$ and $R_v^{(\pm)}$ can be regarded as the "large components" and "small components" of the heavy quark fields $Q^{(\pm)}$ respectively. In this representation, it is independent of the special choice of vector v^μ as long as it satisfies the condition $v^2 = 1$. Note that one should not apply the whole Dirac equation of motion to both quark and antiquark components, otherwise the resulting effective field Lagrangian vanishes. Namely we shall consider the off-mass shell cases with

$$(i\not D_{\parallel} - m_Q) \hat{Q}_v^{(\pm)} + i\not D_{\perp} R_v^{(\pm)} \neq 0, \tag{2.21}$$

$$\bar{\hat{Q}}_v^{(\pm)}(-i\overleftarrow{\not D}_{\parallel} - m_Q) - \bar{R}_v^{(\pm)} i\overleftarrow{\not D}_{\perp} \neq 0. \tag{2.22}$$

With the relation in (2.19) and (2.20), the fields Q and $\bar{Q}$ can be represented by $\hat{Q}_v$ and $\bar{\hat{Q}}_v$ via the following relations

$$Q = \Big[1 + \Big(1 - \frac{i \not{v} v\cdot D + m_Q}{2m_Q}\Big)^{-1} \frac{i\not{D}_\perp}{2m_Q}\Big]\hat{Q}_v \equiv \Big[1 + W_v(\frac{1}{m_Q})\Big]\hat{Q}_v \equiv \hat{\omega}\hat{Q}_v \quad (2.23)$$

$$\bar{Q} = \bar{\hat{Q}}_v\Big[1 - \frac{i\overleftarrow{\not{D}}_\perp}{2m_Q}\Big(1 + \frac{i \not{v} v\cdot \overleftarrow{D} - m_Q}{2m_Q}\Big)^{-1}\Big] \equiv \Big[1 + \overleftarrow{W}_v(\frac{1}{m_Q})\Big]\bar{\hat{Q}}_v \equiv \bar{\hat{Q}}_v\overleftarrow{\hat{\omega}} \quad (2.24)$$

Here for any operator O, the operator $\overleftarrow{O}$ is defined via $\int \kappa \overleftarrow{O}\varphi \equiv -\int \kappa O \varphi$.

To construct the effective field theory of large component $\hat{Q}_v$, one can simply integrate out the small component R_v. At tree level, it is easily shown that this is equivalent to substitute Eqs.(2.23) and (2.24) into the QCD Lagrangian. As a consequence, we arrive at an effective lagrangian for a large component QCD (LCQCD)

$$\mathcal{L}_{QCD} = \mathcal{L}_{light} + \hat{\mathcal{L}}_{Q,v} \quad (2.25)$$

with

$$\begin{aligned}\hat{\mathcal{L}}_{Q,v} &\equiv \bar{Q}(i\not{D} - m_Q)Q|_{Q\to\hat{\omega}\hat{Q}_v} \\ &= \bar{\hat{Q}}_v[i\hat{\not{\mathcal{D}}}_v - m_Q]\hat{Q}_v + \frac{1}{2m_Q}\bar{\hat{Q}}_v(i\overleftarrow{\hat{\not{\mathcal{D}}}}_v - m_Q)\Big(1 - \frac{i \not{v} v\cdot D + m_Q}{2m_Q}\Big)^{-1}(i\not{D}_\perp)\hat{Q}_v \quad (2.26) \\ &\equiv \bar{\hat{Q}}_v[i\hat{\not{\mathcal{D}}}_v - m_Q]\hat{Q}_v + \frac{1}{2m_Q}\bar{\hat{Q}}_v(-i\overleftarrow{\not{D}}_\perp)\Big(1 - \frac{-i \not{v} v\cdot \overleftarrow{D} + m_Q}{2m_Q}\Big)^{-1}(i\hat{\not{\mathcal{D}}}_v - m_Q)\hat{Q}_v\end{aligned}$$

where the operator $i\hat{\not{\mathcal{D}}}_v$ is defined as

$$i\hat{\not{\mathcal{D}}}_v \; = \; i \not{v} v\cdot D + \frac{1}{2m_Q} i\not{D}_\perp\Big(1 - \frac{i \not{v} v\cdot D + m_Q}{2m_Q}\Big)^{-1} i\not{D}_\perp. \quad (2.27)$$

It is easily seen from the definitions (eqs.2.8-2.11) and the commutation relations (2.18) that there is no quark-antiquark coupling in the first term on the rhs. of Eq.(2.26). The second term on the rhs. of Eq.(2.26) arises from the quark-antiquark coupling interactions. To be manifest, one can rewrite the above Lagrangian $\mathcal{L}_{Q,v}$ in the following form

$$\hat{\mathcal{L}}_{Q,v} = \hat{\mathcal{L}}^{(++)}_{Q,v} + \hat{\mathcal{L}}^{(--)}_{Q,v} + \hat{\mathcal{L}}^{(+-)}_{Q,v} + \hat{\mathcal{L}}^{(-+)}_{Q,v} \quad (2.28)$$

with

$$\hat{\mathcal{L}}^{(\pm\pm)}_{Q,v} \; = \; \bar{\hat{Q}}^{(\pm)}_v[i\hat{\not{\mathcal{D}}}_v - m_Q]\hat{Q}^{(\pm)}_v, \quad (2.29)$$

$$\hat{\mathcal{L}}^{(\pm\mp)}_{Q,v} = \frac{1}{2m_Q}\bar{\hat{Q}}^{(\pm)}_v(i\overleftarrow{\hat{\not{\mathcal{D}}}}_v - m_Q)\Big(1 - \frac{i\not{v}v\cdot D + m_Q}{2m_Q}\Big)^{-1}(i\not{D}_\perp)\hat{Q}^{(\mp)}_v \quad (2.30)$$

$$\equiv \frac{1}{2m_Q}\bar{\hat{Q}}^{(\pm)}_v(-i\overleftarrow{\not{D}}_\perp)\Big(1 - \frac{-i\not{v}v\cdot \overleftarrow{D} + m_Q}{2m_Q}\Big)^{-1}(i\hat{\not{\mathcal{D}}}_v - m_Q)\hat{Q}^{(\mp)}_v, \quad (2.31)$$

The operator $\overleftarrow{\hat{\not{\mathcal{D}}}}_v$ in the above equations can be obtained by replacing D^μ with $-\overleftarrow{D^\mu}$ in $\hat{\not{\mathcal{D}}}_v$.

To make $1/m_Q$ expansion, it is useful to remove the large mass term in the effective Lagrangian. We then introduce new field variables Q_v and $\bar{Q}_v$ with the definition

$$Q_v = e^{i\not{v}m_Q v\cdot x}\hat{Q}_v, \qquad \bar{Q}_v = \bar{\hat{Q}}_v e^{-i\not{v}m_Q v\cdot x}. \quad (2.32)$$

Noticing the feature that $\not{v}$ commutes with $\not{D}_\parallel$ but anticommutes with $\not{D}_\perp$, we can rewrite above Lagrangian to be

$$\mathcal{L}_{Q,v} = \mathcal{L}^{(1)}_{Q,v} + \mathcal{L}^{(2)}_{Q,v}$$

$$\mathcal{L}^{(1)}_{Q,v} \equiv \mathcal{L}^{(++)}_{Q,v} + \mathcal{L}^{(--)}_{Q,v} = \bar{Q}_v i\not{\mathcal{D}}_v Q_v, \quad (2.33)$$

$$\mathcal{L}^{(2)}_{Q,v} \equiv \mathcal{L}^{(+-)}_{Q,v} + \mathcal{L}^{(-+)}_{Q,v}$$

$$= \frac{1}{2m_Q}\bar{Q}_v(i\overleftarrow{\not{\mathcal{D}}}_v)e^{2i\not{v}m_Q v\cdot x}\Big(1 - \frac{i\not{v}v\cdot D}{2m_Q}\Big)^{-1}(i\not{D}_\perp)Q_v \quad (2.34)$$

$$\equiv \frac{1}{2m_Q}\bar{Q}_v(-i\overleftarrow{\not{D}}_\perp)\Big(1 - \frac{-i\not{v}v\cdot \overleftarrow{D}}{2m_Q}\Big)^{-1}e^{-2i\not{v}m_Q v\cdot x}(i\not{\mathcal{D}}_v)Q_v, \quad (2.34')$$

with

$$i\not{\mathcal{D}}_v = i\not{v}v\cdot D + \frac{1}{2m_Q}i\not{D}_\perp\Big(1 - \frac{i\not{v}v\cdot D}{2m_Q}\Big)^{-1}i\not{D}_\perp,$$

$$i\overleftarrow{\not{\mathcal{D}}}_v = -i\not{v}v\cdot \overleftarrow{D} + \frac{1}{2m_Q}(-i\overleftarrow{\not{D}}_\perp)\Big(1 - \frac{-i\not{v}v\cdot \overleftarrow{D}}{2m_Q}\Big)^{-1}(-i\overleftarrow{\not{D}}_\perp). \quad (2.35)$$

where we have expressed the effective Lagrangian for the quark-antiquark coupling interactions in terms of two identical formulations (2.34) and (2.34'). One can use either of them, which mainly relies on the convenience for the relevant applications. The factor $e^{\pm 2i\not{v}m_Q v\cdot x}$ arises from the opposite momentum shift for the effective heavy quark and antiquark fields.

The above Lagrangian (eqs.(2.26-2.31) or eqs.(2.33-2.35)) form a basic framework of LCQCD. Note that the resulting effective Lagrangian is a complete one for the

large component of effective heavy quark and antiquark fields, i.e., $Q_v = Q_v^{(+)} + Q_v^{(-)}$, we have only made the field redefinitions and integrated out the small component of effective heavy quark and antiquark fields, i.e., $R_v = R_v^{(+)} + R_v^{(-)}$.

B. Lorentz Invariance of LCQCD

Lorentz invariance is a fundamental requirement of a theory. As the full QCD is established via Lorentz invariance, we shall check how the Lorentz invariance holds in LCQCD which is directly derived from the Lorentz invariant full QCD theory. Before proceeding it would be useful to summarize the procedures for constructing the LCQCD from QCD.

$$\begin{aligned}
\mathcal{L}_Q^{QCD} &= \bar{Q}(i\not{D} - m_Q)Q \\
&\equiv (\bar{\hat{Q}}_v + \bar{R}_v)(i\not{D} - m_Q)(\hat{Q}_v + R_v) \\
&= \bar{\hat{Q}}_v[1 + \overleftarrow{W}_v(\frac{1}{m_Q})](i\not{D} - m_Q)[1 + W_v(\frac{1}{m_Q})]\hat{Q}_v \\
&= \bar{Q}_v e^{i\not{v} m_Q v\cdot x}[1 + \overleftarrow{W}_v(\frac{1}{m_Q})](i\not{D} - m_Q)[1 + W_v(\frac{1}{m_Q})]e^{-i\not{v} m_Q v\cdot x} Q_v \\
&= \bar{Q}_v i\not{v} v\cdot D Q_v + \mathcal{L}_{Q,v}^{QCD}(\frac{1}{m_Q}) \equiv \mathcal{L}_{Q,v}^{QCD}
\end{aligned} \tag{2.36}$$

From this procedure one sees that a crucial point is the introduction of the unit time-like vector v_μ with $v^2 = 1$. As this unit vector is arbitrary, one can introduce another unit vector, say v'_μ with $v'^2 = 1$. Repeating the same procedure, one then obtains an analogous expression for the effective Lagrangian with just replacing v by v', namely

$$\begin{aligned}
\mathcal{L}_Q^{QCD} &= \bar{Q}(i\not{D} - m_Q)Q \\
&\equiv \bar{Q}_v i\not{v} v\cdot D Q_v + \mathcal{L}_{Q,v}^{QCD}(\frac{1}{m_Q}) \equiv \mathcal{L}_{Q,v}^{QCD} \\
&= \bar{Q}_{v'} i\not{v}' v'\cdot D Q_{v'} + \mathcal{L}_{Q,v'}^{QCD}(\frac{1}{m_Q}) \equiv \mathcal{L}_{Q,v'}^{QCD}
\end{aligned} \tag{2.37}$$

which means that the effective Lagrangian in terms of the new variable Q_v with different unit time-like vector v_μ should be equivalent in describing the physics although the new field Q_v with different v_μ represents different field. For convenience we refer to this feature as a super-equivalence of LCQCD Lagrangian.

Keeping this point in mind, we now return to discuss the Lorentz invariance of $\mathcal{L}_{Q,v}^{QCD}$. The full QCD theory is invariant under the Lorentz transformation

$$x^\mu \to \Lambda^\mu_\nu x^\nu, \qquad Q(x) \to Q'(x') = D(\Lambda)Q(\Lambda^{-1}x) \tag{2.38}$$

with

$$\Lambda^\mu_\nu = g^\mu_\nu + \omega^\mu_\nu, \qquad D(\Lambda) = e^{-\frac{i}{4}\sigma_{\mu\nu}\omega^{\mu\nu}}$$

From the definition of Q_v , it is not difficult to show that under the above transformation the Q_v transforms as

$$\begin{aligned}
Q_v(x) \quad & \to Q'_{v'}(x') \\
& = e^{i\not{v} m_Q v\cdot x}\frac{1+\not{v}}{2}D(\Lambda)Q^{(+)}(\Lambda^{-1}x) + e^{i\not{v} m_Q v\cdot x}\frac{1-\not{v}}{2}D(\Lambda)Q^{(-)}(\Lambda^{-1}x) \\
& = D(\Lambda)[D^{-1}(\Lambda)e^{i\not{v} m_Q v\cdot x}D(\Lambda)][D^{-1}(\Lambda)\frac{1+\not{v}}{2}D(\Lambda)]Q^{(+)}(\Lambda^{-1}x) \\
& +D(\Lambda)[D^{-1}(\Lambda)e^{i\not{v} m_Q v\cdot x}D(\Lambda)][D^{-1}(\Lambda)\frac{1-\not{v}}{2}D(\Lambda)]Q^{(-)}(\Lambda^{-1}x) \\
& = D(\Lambda)e^{i\not{v}' m_Q v'\cdot x'}\frac{1+\not{v}'}{2}Q^{(+)}(x') + D(\Lambda)e^{i\not{v}' m_Q v'\cdot x'}\frac{1-\not{v}'}{2}Q^{(-)}(x') \\
& = D(\Lambda)[e^{i\not{v}' m_Q v'\cdot x'}\frac{1+\not{v}'}{2}Q^{(+)}(x') + e^{i\not{v}' m_Q v'\cdot x'}\frac{1-\not{v}'}{2}Q^{(-)}(x')] \\
& = D(\Lambda)Q_{v'}(x')
\end{aligned} \tag{2.39}$$

with

$$v' = \Lambda^{-1}v, \qquad x' = \Lambda^{-1}x$$

Here we have used the transformation property

$$D^{-1}(\Lambda)\not{v}D(\Lambda) = \not{v}' \tag{2.40}$$

which indicates that under the Lorentz transformation the vector v_μ should transform like the coordinate x_μ. The Lagrangian $\mathcal{L}_{Q,v}^{QCD}$ becomes, under the Lorentz transformation, to be

$$\mathcal{L}_{Q,v}^{QCD} \to \mathcal{L}_{Q,v'}^{QCD} = \bar{Q}_{v'}i\not{v}'v'\cdot DQ_{v'} + \mathcal{L}_{Q,v'}^{QCD}(\frac{1}{m_Q}) \tag{2.41}$$

The Lorentz invariance is recovered by the super-equivalence of LCQCD Lagrangian as shown explicitly above.

Note that when the effective Lagrangian is truncated in the expansion of $1/m_Q$ to only finite terms, the Lorentz invariance will be broken down.

C. Quark-antiquark Couplings and Their Decouple Conditions

Here we would like to address two important issues: Firstly, in the effective lagrangian of LCQCD the quark-antiquark coupling terms appear naturally as long as one treats the quark and antiquark fields on the same footing in a symmetric way. This is actually the basic feature of quantum field theory. It is also explicitly seen that in the infinite mass limit, the quark and antiquark fields decouple each other, namely quark-antiquark coupling terms approach to vanish. In general, we will show below that such quark-antiquark coupling terms will provide important contributions starting from the order of $1/m_Q$. Therefore in order to calculate correctly the $1/m_Q$ corrections arising from the finite mass of heavy quark, one should take into account the contributions from the terms $\mathcal{L}_{Q,v}^{(\pm\mp)}$ and $\mathcal{L}_{Q,v}^{(--)}$ for the processes involving the heavy quark $Q_v^{(+)}$ or from the terms $\mathcal{L}_{Q,v}^{(\pm\mp)}$ and $\mathcal{L}_{Q,v}^{(++)}$ for the ones involving the heavy antiquark $Q_v^{(-)}$. This is because through virtual antiquark or quark field exchanges they will produce $1/m_Q$ corrections via quantum effects. Only in the infinite mass limit, the quark-antiquark coupling terms decouple in the sense of effective quantum field theory, namely

$$\hat{\mathcal{L}}_{Q,v} \to \hat{\mathcal{L}}_{Q,v}^{(m_Q\to\infty)} \; = \; \bar{\hat{Q}}_v^{(+)}[i\not{D}_{\parallel} - m_Q]\hat{Q}_v^{(+)} + \bar{\hat{Q}}_v^{(-)}[i\not{D}_{\parallel} - m_Q]\hat{Q}_v^{(-)} \tag{2.42}$$

Secondly, one may not use the whole Dirac equation of motion either for quark component or antiquark component. Otherwise, either the quark field or antiquark field will decouple from effective Lagrangian, which leads the effective heavy quark and antiquark fields to be treated not on the same footing via a symmetric way. To be explicit, it is easily seen that if one further uses the Dirac equation of motion for the effective heavy antiquark field, namely,

$$(i\not{D}_{\parallel} - m_Q)\hat{Q}_v^{(-)} + i\not{D}_{\perp} R_v^{(-)} = 0, \quad \text{i.e.} \quad (i\hat{\not{\mathcal{D}}}_v - m_Q)\hat{Q}_v^{(-)} = 0 \tag{2.43}$$

then the heavy quark effective Lagrangian is reduced to be

$$\hat{\mathcal{L}}_{Q,v} \to \hat{\mathcal{L}}_{Q,v}^{(++)} = \bar{\hat{Q}}_v^{(+)}[i\hat{\not{\mathcal{D}}}_v - m_Q]\hat{Q}_v^{(+)} \tag{2.44}$$

which recovers the widely used heavy quark effective Lagrangian for effective heavy quark field $\hat{Q}_v^{(+)}$. Similarly, if further adopting the Dirac equation of motion for

effective heavy quark field

$$(i\not{D}_{\parallel} - m_Q)\hat{Q}_v^{(+)} + i\not{D}_{\perp} R_v^{(+)} = 0, \quad \text{i.e.} \quad (i\hat{\not{D}}_v - m_Q)\hat{Q}_v^{(+)} = 0 \tag{2.45}$$

one arrives at the heavy quark effective Lagrangian

$$\hat{\mathcal{L}}_{Q,v} \to \hat{\mathcal{L}}_{Q,v}^{(--)} = \bar{\hat{Q}}_v^{(-)}[i\hat{\not{D}}_v - m_Q]\hat{Q}_v^{(-)} \tag{2.46}$$

which reproduces the widely used heavy quark effective Lagrangian for effective heavy antiquark field $\hat{Q}_v^{(-)}$.

From the above analyzes, it is not difficult to find out an implicit assumption made in the widely used heavy quark effective theory which describes either quark field or antiquark field and contains no quark-antiquark coupling terms. It also implies that the assumption of particle and antiparticle numbers being separately conserved made in the derivation of widely used heavy quark effective Lagrangian is somehow equivalent to the case that the Dirac equation of motion has been imposed for either antiquark field or quark field.

We can now conclude that in the case of $|\mathbf{p}| << E + m_Q$ but without imposing Dirac equation of motion for either quark field or antiquark field, QCD shall be described by a large component QCD (LCQCD) with containing both large component quark and antiquark fields in the heavy quark effective lagrangian.

D. HQEFT from LCQCD

For any practical application of LCQCD to heavy quark systems, one may further expand the LCQCD Lagrangian into a series in powers of the inverse heavy quark mass. In order to correctly perform such an expansion, one must keep an eye on the physical conditions relating to the concrete physical systems. In general, the "longitudinal" and "transverse" residual momenta of heavy quarks can be at different powers for different physical systems or processes. Correspondingly, the operator $\not{D}_{\parallel}$ and $\not{D}_{\perp}$ may be treated as operators in different power counting in the $1/m_Q$ expansion. We will consider below two interesting cases which are corresponding to two typical conditions.

One interesting case is for the heavy quark being nearly on-mass shell. That may happen for heavy quarks within the heavy quarkonia system. This implies the operator relation for the $1/m_Q$ expansion

$$i \not{v} \cdot D = -\frac{(i\not{D}_\perp)^2}{2m_Q} + O\Big(\frac{1}{m_Q^2}\Big). \tag{2.47}$$

With this condition, it is not difficult to see that the quark-antiquark coupling terms $\mathcal{L}^{(2)}_{Q,v}$ are suppressed by the higher order terms of $1/m_Q$. Thus the quark-quark and antiquark-antiquark coupling terms $\mathcal{L}^{(1)}_{Q,v}$ become dominant. This case may be dealt with like a nonrelativistic QCD (NRQCD). Such a case was also discussed in ref.[18] and applied to heavy quark pair creations near the threshold.

Another interesting case is for heavy quark being slightly off-mass shell. The typical case is considered for the heavy hadron containing a single heavy quark. The magnitude of the off-mass shell is usually thought to be at the order of binding energy with the off-mass shell condition

$$\frac{p^2 - m_Q^2}{2m_Q} \sim \bar{\Lambda},$$

Taking $p = m_Q v + k$ with $v = (1,0,0,0)$, one has

$$k^0 \sim |\mathbf{k}| \sim \bar{\Lambda}, \qquad \text{i.e.} \quad v \cdot k \sim |k_\perp| \sim \bar{\Lambda}$$

Here $\bar{\Lambda} \sim 2\Lambda_{QCD} \sim 500 \sim 600$ MeV is the typical binding energy of heavy hadrons. In the operator basis, the above condition is corresponding to:

$$\langle iD_\parallel \rangle \sim \langle iD_\perp \rangle \sim \bar{\Lambda}, \tag{2.48}$$

which implies that the operators corresponding to "longitudinal" and "transverse" residual momenta are taken to be at the same order of power counting in the $1/m_Q$ expansion. In this case, the quark-antiquark coupling terms $\mathcal{L}^{(2)}_{Q,v}$ are at the order of $O\big(1/m_Q\big)$. In this sense, LCQCD is treated as a heavy quark effective field theory (HQEFT) via $1/m_Q$ expansion.

To see explicitly the effects of antiquark field, i.e., the contributions from quark-antiquark coupling terms, we may integrate out the effective heavy antiquark fields, which is equivalent, at the tree level, to adopt the following relations between quark

and antiquark field to eliminate the antiquark field in the effective Lagrangian of LCQCD

$$(\hat{\not{\mathcal{D}}}_v - m_Q)\hat{Q}_v^{(-)} + [i\not{D}_\perp(m_Q - i\not{v}\cdot D)^{-1}(i\hat{\not{\mathcal{D}}}_v - m_Q)]\hat{Q}_v^{(+)} = 0, \quad (2.49)$$

$$\bar{\hat{Q}}_v^{(-)}(\overleftarrow{\hat{\not{\mathcal{D}}}}_v - m_Q) + \bar{\hat{Q}}_v^{(+)}[i\overleftarrow{\not{D}}_\perp(m_Q - i\not{v}\cdot\overleftarrow{D})^{-1}(i\overleftarrow{\hat{\not{\mathcal{D}}}}_v - m_Q)] = 0, \quad (2.50)$$

or

$$\hat{Q}_v^{(-)} = -(\hat{\not{\mathcal{D}}}_v - m_Q)^{-1}[i\not{D}_\perp(m_Q - i\not{v}\cdot D)^{-1}(i\hat{\not{\mathcal{D}}}_v - m_Q)]\hat{Q}_v^{(+)}, \quad (2.51)$$

$$\bar{\hat{Q}}_v^{(-)} = -\bar{\hat{Q}}_v^{(+)}[i\overleftarrow{\not{D}}_\perp(m_Q - i\not{v}\cdot\overleftarrow{D})(i\overleftarrow{\hat{\not{\mathcal{D}}}}_v - m_Q)](\overleftarrow{\hat{\not{\mathcal{D}}}}_v - m_Q)^{-1}. \quad (2.52)$$

The resulting effective Lagrangian for quark field has the following form

$$\mathcal{L}_{eff}^{(++)} = \mathcal{L}_{Q,v}^{(++)} + \tilde{\mathcal{L}}_{Q,v}^{(++)}, \quad (2.53)$$

The second part $\tilde{\mathcal{L}}_{Q,v}^{(++)}$ comes from the contributions of integrating out the effective heavy quark antiquark field. Its explicit form is found to be

$$\begin{aligned}\tilde{\mathcal{L}}_{Q,v}^{(++)} &= \langle \mathcal{L}_{Q,v}^{(--)} + \mathcal{L}_{Q,v}^{(+-)} + \mathcal{L}_{Q,v}^{(-+)}\rangle|_{\hat{Q}_v^{(-)}\to\hat{Q}_v^{(+)}} \quad (2.54)\\ &= -\bar{\hat{Q}}_v^{(+)} i\not{D}_\perp(m_Q - i\not{v}\cdot D)^{-1} i\not{D}_\perp(m_Q - i\not{v}\cdot D)^{-1}(i\hat{\not{\mathcal{D}}}_v - m_Q)\hat{Q}_v^{(+)}\end{aligned}$$

After removing the large mass term in the above effective Lagrangian, we then obtain the following compact form

$$\mathcal{L}_{eff}^{(++)} = \bar{Q}_v^{(+)} i\not{\mathcal{D}}_v \frac{1}{i\not{v}\cdot D} i\not{\mathcal{D}}_v Q_v^{(+)} \quad (2.55)$$

which may be rewritten into the following explicit form

$$\begin{aligned}\mathcal{L}_{eff}^{(++)} &= \bar{Q}_v^{(+)} i\not{v}\cdot D Q_v^{(+)}\\ &+2\cdot\frac{1}{2m_Q}\bar{Q}_v^{(+)}(i\not{D}_\perp)\Big(1 - \frac{i\not{v}\cdot D}{2m_Q}\Big)^{-1}(i\not{D}_\perp)Q_v^{(+)}\\ &+\frac{1}{4m_Q^2}\bar{Q}_v^{(+)}(i\not{D}_\perp)\Big(1 - \frac{i\not{v}\cdot D}{2m_Q}\Big)^{-1}(i\not{D}_\perp)\\ &\times\frac{1}{i\not{v}\cdot D}(i\not{D}_\perp)\Big(1 - \frac{i\not{v}\cdot D}{2m_Q}\Big)^{-1}(i\not{D}_\perp)Q_v^{(+)}. \quad (2.56)\end{aligned}$$

It must be noted that the power counting order of $1/m_Q$ expansion for the third term in above equation will depend on the order of power counting for the operators $\not{v}\cdot D$ and $\not{D}_\perp$.

We now return to discuss the two interesting cases considered above. For the first case with the condition (2.47), the third term in (2.56) is actually at the same order of power counting as the second term in (2.56). This is because the "longitudinal" operator $\not{v} \cdot D$ is suppressed by $1/m_Q$ in comparison with the "transverse" operator $\not{D}_\perp$. Substituting to the condition (2.47) to the third term of the above effective Lagrangian (2.56), we then arrive at the following effective Lagrangian expanding in terms of the $1/m_Q$

$$\mathcal{L}_{eff}^{(++)} = \bar{Q}_v^{(+)} \Big\{ i \not{v} \cdot D + \frac{1}{2m_Q} (i\not{D}_\perp)^2 \Big\} Q_v^{(+)} + O\Big(\frac{1}{m_Q^2}\Big). \tag{2.57}$$

Taking $v = (1, 0, 0, 0)$ and $A_0 = 0$, one has

$$\mathcal{L}_{eff}^{(++)} = \bar{Q}_v^{(+)} \Big\{ i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m_Q} \Big\} Q_v^{(+)} + O\Big(\frac{1}{m_Q^2}\Big). \tag{2.58}$$

which recovers the effective Lagrangian in NRQCD.

For the second case with the condition (2.48), the third term in (2.56) can now be treated as $1/m_Q^2$ order and the second term in (2.56) as $1/m_Q$ order. Thus the Lagrangian (2.56) can be expanded in terms of $1/m_Q$ into the following form

$$\mathcal{L}_{eff}^{(++)} = \mathcal{L}_{eff}^{(0)} + \mathcal{L}_{eff}^{(1/m_Q)} \tag{2.59}$$

with

$$\mathcal{L}_{eff}^{(0)} = \bar{Q}_v^{(+)} (i \not{v} \cdot D) Q_v^{(+)}, \tag{2.60}$$

$$\begin{aligned}\mathcal{L}_{eff}^{(1/m_Q)} &= 2\ \frac{1}{2m_Q} \bar{Q}_v^{(+)} (i\not{D}_\perp)^2 Q_v^{(+)} + 2\ \frac{1}{4m_Q^2} \bar{Q}_v^{(+)} i\not{D}_\perp (i \not{v} \cdot D) i\not{D}_\perp Q_v^{(+)} \\ &+ \frac{1}{4m_Q^2} \bar{Q}_v^{(+)} (i\not{D}_\perp)^2 \frac{1}{i \not{v} \cdot D} (i\not{D}_\perp)^2 Q_v^{(+)} + O\Big(\frac{1}{m_Q^3}\Big),\end{aligned} \tag{2.61}$$

where $\mathcal{L}_{eff}^{(0)}$ is the leading term and possesses the spin-flavor symmetry. The second part $\mathcal{L}_{eff}^{(1/m_Q)}$ contains the remaining terms which are the spin-flavor symmetry breaking ones suppressed by $1/m_Q$. Here we have used the condition for the operators: $(i\not{D}_\perp)^2/2m_Q \ll i \not{v} \cdot D$ which always holds for the typical case that $\langle iD_\parallel \rangle \sim \langle iD_\perp \rangle \sim \bar{\Lambda} \ll m_Q$ in the heavy quark expansion. In this case, the third term in the rhs. of Eq.(2.61) is now regarded as at the order of $1/m_Q^2$ and the

‘nonlocal’ factor $1/i \not{v} v \cdot D$ actually arises as a general propagator due to the virtual effective antiquark exchange in HQEFT (see below). The first term in the rhs. of Eq.(2.61) represents the total $1/m_Q$ order correction to the leading order Lagrangian (2.60). One may notice that there is a factor of two in the first two terms of Eq.(2.61), which can now be well understood from the $1/m_Q$ expansion in HQEFT. It clearly arises from the antiquark contributions when the heavy quark is considered to be slightly off-mass shell with $\langle iD_{\|}\rangle \sim \langle iD_{\perp}\rangle \sim \bar{\Lambda}$, which satisfies the condition in the momentum space that $v \cdot k \sim \bar{\Lambda} > |k_{\perp}^2|/2m_Q$ (i.e., $|k_{\perp}| < \sqrt{2\bar{\Lambda}m_Q}$).

It is seen that the treatment of HQEFT should differ from the one of NRQCD. Besides the explicit difference for the factor of two in the $1/m_Q$ and $1/m_Q^2$ terms, the propagators in the treatment of effective field theories for NRQCD and HQEFT must also be different. The propagator in NRQCD should take the following form

$$\frac{i}{\not{v} v \cdot k + \frac{1}{2m_Q}(\not{k}_{\perp})^2} \tag{2.62}$$

as the two terms in the denominator are compatible for the nearly on-mass shell condition. Whereas the propagator in HQEFT is mainly governed by the “longitudinal” part and will be shown late on to have the following simple form

$$\frac{i}{\not{v} v \cdot k} \tag{2.63}$$

It is necessary to address that only with the above considerations one is able to obtain consistent effective field theories within the framework of LCQCD derived directly from full QCD. Obviously, the above analyzes also make HQEFT different from the treatment of the widely used heavy quark effective theory in which the quark and antiquark fields are not treated on the same footing in a symmetric way in the widely used heavy quark effective theory.

We now arrive at the conclusion that when applying LCQCD to the physical system in which the single heavy quark within a hadron is slightly off-mass shell with the typical energy region that $\langle iD_{\|}\rangle \sim \langle iD_{\perp}\rangle \sim \bar{\Lambda}$, namely $v \cdot k \sim \bar{\Lambda} \gg |k_{\perp}^2|/2m_Q$ (i.e., $|k_{\perp}| \ll \sqrt{2\bar{\Lambda}m_Q}$), only HQEFT described above becomes appropriate and consistent. As the effective heavy quark and antiquark fields in such a kind of HQEFT are treated on the same footing in a fully symmetric way, which also fits

to the spirit of quantum field theory as the original QCD does, it then allows us to establish a complete theoretical framework of HQEFT in the sense of effective quantum field theory. This comes to the main task in the following sections.

Note that one should avoid some misleading questions raised from the usual heavy quark effective theory for the quark-antiquark coupling, such as *no such terms could appear in the effective Lagrangian because the soft gluon-quark-antiquark vertex does not conserve momentum.* As we have shown in the subsection (2.1) that our effective Lagrangian is formally derived from the full QCD by just integrating out the small components of heavy quark fields, all terms must exist in the effective Lagrangian. The momentum conservation only constraints that a soft gluon cannot create a heavy quark-antiquark pair, which can explicitly be seen from the factor $exp(-i2m_Q.x)$ in the quark-antiquark coupling terms when introducing the residual momentum, but it doesn't mean that such quark-antiquark coupling terms cannot appear in the effective Lagrangian as the soft gluon-quark-antiquark vertex can always be momentum conserved as long as the quark and antiquark fields are slightly off-mall shell with one of them is virtual. In this case, both "quark" and "anti-quark" fields actually carry either positive energy or negative energy. Nevertheless, when the virtual "anti-quark" field carries positive energy, it is suppressed by $1/m_Q$. Similarly, when virtual "quark" field carries negative energy, it is also suppressed by $1/m_Q$. In fact, even in the full QCD soft gluon cannot create the quark-antiquark pairs but the quark-antiquark coupling term remains there. It is easily shown that the quark-antiquark coupling terms with soft gluon can have contributions to the quark-antiquark scattering via t-channel, it can also provide important effects for the quark $\rightarrow$ quark transition via antiquark field mediation or antiquark $\rightarrow$ antiquark transition via quark field mediation. The latter is interesting for studying heavy quark decays in the HQEFT. This is what we are going to demonstrate in details in this paper, it is actually the natural features of quantum field theory.

III. QUANTIZATION OF HQEFT

A. Quantum Generators of Poincare Group

Let us first consider the case of a free field theory for a single quark at infinite mass limit. The effective Lagrangian for this case is simple

$$\mathcal{L}^{(00)}_{Q,v} = \bar{Q}_v i\not{v} v \cdot \partial Q_v \tag{3.1}$$

The equation of motion reads

$$i\not{v} v \cdot \partial Q_v = 0 \tag{3.2}$$

Since the field $Q_v(x)$ and $\bar{Q}_v(x)$ concern the additional arbitrary unit time-like vector v_μ, the canonical quantization is not quite clear for it, we cannot directly follow the procedure of canonical quantization and will adopt an alternative procedure. Firstly it is assumed that $Q_v(x)$ and $\bar{Q}_v(x)$ are quantized operators acting in an Hilbert space, we then look for the conditions under which Poincare group generators constructed from Noether's theorem will lead the fields to transform according to the required transformation laws. Since these generators and also all observable quantities are expressed in terms of the dynamical variables $Q_v(x)$ and $\bar{Q}_v(x)$, we shall verify that these observables do commute at space-like separations and the theory is ensured to be covariant.

Consider an infinitesimal x-dependent translation $x \to x + a(x)$, thus

$$\begin{aligned} \delta Q_v = \delta a^\mu \partial_\mu Q_v, \qquad \delta\bar{Q}_v = \bar{Q}_v \overleftarrow{\partial_\mu} \delta a^\mu \\ \delta[\partial_\mu Q_v] = \delta a^\nu \partial_\nu \partial_\mu Q_v + \partial_\mu[\delta a^\nu]\partial_\nu Q_v \end{aligned} \tag{3.3}$$

The corresponding variation of the action after an integration by parts is

$$\delta I_v = \int \partial_\mu [g^{\mu\nu} \bar{Q}_v i\not{v} v \cdot \partial Q_v - \bar{Q}_v i\not{v} v^\mu \partial^\nu Q_v] \delta a_\nu \tag{3.4}$$

From the vanishing of δI_v for arbitrary $\delta a^\nu(x)$, and using the equation of motion, we deduce that the energy momentum flow is described by the canonical tensor

$$\Theta^{\mu\nu}_v = \bar{Q}_v i\not{v} v^\mu \partial^\nu Q_v \tag{3.5}$$

which satisfies the conservation law

$$\partial_\mu \Theta_v^{\mu\nu} = 0 \tag{3.6}$$

Consider now an infinitesimal homogeneous Lorentz transformation under which we have

$$Q_v(x) \to Q'_{v'}(x') = D(\Lambda) Q_{v'}(\Lambda^{-1}x) = Q_{v'}(x) + \delta Q_{v'}(x) \tag{3.7}$$

with

$$\delta Q_{v'}(x) = -\frac{i}{2}\delta\omega_{\alpha\beta} L^{\alpha\beta} Q_{v'}(x), \qquad L^{\alpha\beta} = \frac{\sigma^{\alpha\beta}}{2} + i(x^\alpha \partial^\beta - x^\beta \partial^\alpha)$$

and

$$v' = \Lambda^{-1} v = v + \delta v, \qquad \delta \not{v}' = \frac{i}{2}\delta\omega_{\alpha\beta}[i(v'^\alpha \gamma^\beta - v'^\beta \gamma^\alpha)] \tag{3.8}$$

The corresponding variation of the action is

$$\begin{aligned} I_v \to I'_v = & \int d^4x [\bar{Q}_{v'} i \not{v}' v' \cdot \partial Q_{v'} + \delta \bar{Q}_{v'} i \not{v}' v' \cdot \partial Q_{v'} \\ & + \bar{Q}_{v'} i \not{v}' v' \cdot \partial \delta Q_{v'} + \bar{Q}_{v'} i \delta \not{v}' v' \cdot \partial Q_{v'} \\ & + \bar{Q}_{v'} i \not{v}' \delta (v' \cdot \partial) Q_{v'}] \end{aligned} \tag{3.9}$$

Thus we find that as a variation around the stationary point

$$\delta I_{v'} = -\int d^4x (\partial_\mu J_{v'}^{\mu,\alpha\beta}) \frac{\omega_{\alpha\beta}}{2} = 0$$

with

$$J_{v'}^{\mu,\alpha\beta} = v'^\mu \bar{Q}_{v'} \not{v}' L^{\alpha\beta} Q_{v'} = x^\alpha \Theta_{v'}^{\mu\beta} - x^\beta \Theta_{v'}^{\mu\alpha} + v'^\mu \bar{Q}_{v'} \not{v}' \frac{\sigma^{\alpha\beta}}{2} Q_{v'} \tag{3.10}$$

which has an orbital and a spin part, and satisfies the conservation law

$$\partial_\mu J_{v'}^{\mu,\alpha\beta} = 0 \tag{3.11}$$

One can also directly check this conservation law.

Thus the quantum generators of the Poincare group would be yielded by the space integrals of $\Theta_v^{0\mu}$ and $J_{v'}^{0,\alpha\beta}$

$$K_v^\mu = \int d^3x \Theta_v^{0\mu}, \qquad J_{v'}^{\mu\nu} = \int d^3x J_{v'}^{0,\mu\nu} \tag{3.12}$$

which will be used to find a commutation prescription for the effective fields Q_v and $\bar{Q}_v$.

B. Anticommutations and Georgi's Velocity Super-selection Rule

To establish the commutation prescription for the effective fields, we expand the operators Q_v and $\bar{Q}_v$ in terms of the c-number plane wave solutions of the equation of motion eq.(3.2) with operator-valued amplitudes b, $b^\dagger$, d, $d^\dagger$

$$\begin{aligned} Q_v(x) &= \int \frac{d^3k}{(2\pi)^3 v^0} \sum_s [b_v(k,s)u(v,s)e^{-ik\cdot x} + d_v^\dagger(k,s)v(v,s)e^{ik\cdot x}] \\ \bar{Q}_v(x) &= \int \frac{d^3k}{(2\pi)^3 v^0} \sum_s [b_v^\dagger(k,s)\bar{u}(v,s)e^{ik\cdot x} + d_v(k,s)\bar{v}(v,s)e^{-ik\cdot x}] \end{aligned} \tag{3.13}$$

where spinors u and v satisfy

$$\not{v}u(v,s) = u(v,s), \qquad \not{v}v(v,s) = -v(v,s) \tag{3.14}$$

and normalization conditions

$$\begin{aligned} &\bar{u}(v,s)u(v,s') = \delta_{ss'}, \qquad \bar{v}(v,s)v(v,s') = -\delta_{ss'} \\ &\bar{u}(v,s)v(v,s') = \bar{v}(v,s)u(v,s') = 0 \end{aligned} \tag{3.15}$$

Here k^0 is defined through $k \cdot v = 0$ due to the equation of motion. The operators b and d must satisfy commutation rules such that the Poincare group generators transform the fields according to the required transformation laws as indicated from above subsection.

$$\begin{aligned} Q_v(x+a) &= e^{iK_v\cdot a}Q_v(x)e^{-iK_v\cdot a} \\ Q'_{v'}(x') &= e^{iJ^{\mu\nu}_{v'}\omega_{\mu\nu}/2}Q_v(x)e^{-iJ^{\mu\nu}_{v'}\omega_{\mu\nu}/2} \end{aligned} \tag{3.16}$$

with $v' = \Lambda^{-1}v$ and $x' = \Lambda^{-1}x$. In the differential form, they can be rewritten as

$$\partial^\mu Q_v(x) = i[K_v^\mu, Q_v(x)], \qquad \partial^\mu \bar{Q}_v(x) = i[K_v^\mu, \bar{Q}_v(x)] \tag{3.17}$$

$$L^{\mu\nu}Q_{v'}(x) = -[J^{\mu\nu}_{v'}, Q_v(x)], \qquad \bar{Q}_{v'}(x)\overleftarrow{L}^{\mu\nu} = -[J^{\mu\nu}_{v'}, \bar{Q}_v(x)] \tag{3.18}$$

Let us first consider the requirement of translational invariance. Expressing K_v^μ with the decomposition eqs.(3.5), (3.12) and (3.13)

$$K_v^\mu = \int d^3x \Theta_v^{0\mu} = \int \frac{d^3x}{(2\pi)^3 v^0} k^\mu \sum_s [b_v^\dagger(k,s)b_v(k,s) - d_v(k,s)d_v^\dagger(k,s)] \tag{3.19}$$

Note that one has to subtract the vacuum contribution in this expression. If the vacuum is defined in such a way that $b_v(k,s)|0>= d_v(k,s)|0>= 0$, we see that the operators b and d have to quantize according to anticommutators rather than commutators, otherwise b particles and d particles contribute with opposite signs to the energy and the theory would not admit a stable ground state.

To obtain explicitly the commutation rule, we express the differential form of the translational invariance in eq.(3.17) in the momentum space. It is satisfied provided

$$\begin{aligned}
&[K_v^\mu, b_v(k,s)] = -k^\mu b_v(k,s), \qquad [K_v^\mu, d_v(k,s)] = -k^\mu d_v(k,s) \\
&[K_v^\mu, b_v^\dagger(k,s)] = k^\mu b_v^\dagger(k,s), \qquad [K_v^\mu, d_v^\dagger(k,s)] = k^\mu d_v^\dagger(k,s)
\end{aligned} \tag{3.20}$$

Using the explicit form of K_v^μ, one can find that to ensure the correct interpretation of energy and momentum the operators should satisfy the anticommutation relations

$$\begin{aligned}
\{b_v(k',s'), b_v^\dagger(k,s)\} &= (2\pi)^3 v^0 \delta^3(\overrightarrow{k} - \overrightarrow{k}')\delta_{s,s'} \\
\{d_v(k',s'), d_v^\dagger(k,s)\} &= (2\pi)^3 v^0 \delta^3(\overrightarrow{k} - \overrightarrow{k}')\delta_{s,s'}
\end{aligned} \tag{3.21}$$

and all other anticommutators vanish.

We now turn to the requirement of the Lorentz transformation invariance. Expressing also $J_{v'}^{\mu\nu}$ with the decomposition eqs. (3.10), (3.12) and (3.13)

$$\begin{aligned}
J_{v'}^{\mu\nu} = \int d^3x J_{v'}^{0;\mu\nu} = \quad & \int \frac{d^3k}{(2\pi)^3 v'^0} \sum_s \sum_{s'} [b_{v'}^\dagger(k,s) b_{v'}(k,s') \bar{u}(v',s) \not{v}' L^{\mu\nu} u(v',s') \\
& + b_{v'}^\dagger(k,s) d_{v'}(-k,s') \bar{u}(v',s) \not{v}' L^{\mu\nu} v(v',s') \\
& + d_{v'}(k,s) b_{v'}(-k,s') \bar{v}(v',s) \not{v}' L^{\mu\nu} u(v',s') \\
& + d_{v'}(k,s) d_{v'}^\dagger(k,s') \bar{v}(v',s) \not{v}' L^{\mu\nu} v(v',s')]
\end{aligned} \tag{3.22}$$

From eq.(3.18), it is seen that the commutations involve operators with different velocities. By noticing the following facts

$$\begin{aligned}
&\bar{u}(v,s)\not{v} = \bar{u}(v,s), \qquad \bar{v}(v,s)\not{v} = -\bar{v}(v,s) \\
&\sum_s u(v,s)\bar{u}(v,s) = \frac{1+\not{v}}{2}, \qquad \sum_s v(v,s)\bar{v}(v,s) = -\frac{1-\not{v}}{2}
\end{aligned} \tag{3.23}$$

as well as the indication of commutation rules obtained from the translational invariance , it is not difficult to find that the anticommutators of operators with different

velocity must satisfy

$$
\begin{aligned}
\{b_{v'}(k',s'),b_v^\dagger(k,s)\} &= (2\pi)^3 v^0\delta^3(\overrightarrow{k}-\overrightarrow{k}')\delta_{ss'}\delta_{vv'}\\
\{d_{v'}(k',s'),d_v^\dagger(k,s)\} &= (2\pi)^3 v^0\delta^3(\overrightarrow{k}-\overrightarrow{k}')\delta_{ss'}\delta_{vv'}
\end{aligned}
\tag{3.24}
$$

and all other anticommutators vanish. Where $\delta_{vv'}$ is a Kronecker delta function, i.e. $\delta_{vv'}=1$ if $v=v'$ and $\delta_{vv'}=0$ if $v\neq v'$. This may be mentioned, as it was first imposed by Georgi[12], to be Georgi's velocity super-selection rule which is in principle a consequence of Lorentz invariance.

From the above commutation rules of operators, we arrive at the commutation rule for the effective fields

$$
\{\Pi_v^\alpha(x),Q_{v'}^\beta(x')\}|_{x0=x_0'} = i\delta_{\alpha\beta}\delta_{vv'}\delta^3(\overrightarrow{x}-\overrightarrow{x}')
\tag{3.25}
$$

where Π_v is the momentum conjugate to the field Q_v and given by

$$
\Pi_v = \frac{\delta L}{\delta(\partial_0 Q_v)} = \bar{Q}_v i\not{v}v^0
\tag{3.26}
$$

This commutation rule of effective fields shows how the HQEFT of LCQCD is quantized canonically.

We can now use the Wick products to define correctly the total energy momentum since when reordering the creation operators to the left of the annihilation ones, a sign corresponding to the parity of the permutation must be introduced, namely

$$
\begin{aligned}
K_v^\mu &= \int\frac{d^3k}{(2\pi)^3v^0}k^\mu\sum_s : b_v^\dagger(k,s)b_v(k,s)-d_v(k,s)d_v^\dagger(k,s):\\
&= \int\frac{d^3k}{(2\pi)^3v^0}k^\mu\sum_s [b_v^\dagger(k,s)b_v(k,s)+d_v^\dagger(k,s)d_v(k,s)]
\end{aligned}
\tag{3.27}
$$

which leads to a sum of positive contributions to the energy of a quantum state. We are now in the position to construct the corresponding Hilbert and Fock space for HQEFT of LCQCD.

C. Hilbert and Fock Space of HQEFT

Let us first consider single-particle states. The necessary smearing in momentum space is implicitly understood. For a given four-velocity v_μ and four-momentum k_μ

there is a fourfold degeneracy. Denote the corresponding states $|I>$ $(I = 1, 2, 3, 4)$ with

$$\begin{aligned} &|1>= b_v^\dagger(k,+)|0>, \qquad |2>= b_v^\dagger(k,-)|0> \\ &|3>= d_v^\dagger(k,+)|0>, \qquad |4>= d_v^\dagger(k,-)|0> \end{aligned} \tag{3.28}$$

They satisfy

$$K_v^\mu |I>= k^\mu |I> \tag{3.29}$$

In order to distinguish these states, let us look for observables commutating with K_v^μ. One of these observables is the charge which is characterized by the phase transformations of the fields

$$Q_v \to e^{i\alpha} Q_v, \qquad \bar{Q}_v \to e^{-i\alpha} \bar{Q}_v \tag{3.30}$$

The invariance of the Lagrangian under this transformation leads to a conserved current

$$J^\mu = v^\mu \bar{Q}_v \not{v} Q_v \tag{3.31}$$

The space integral of J^0 represents the quantum generators of this transformation

$$\begin{aligned} Q_c &= \int d^3x J^0 = \int d^3x v^0 : \bar{Q}_v \not{v} Q_v : \\ &= \int \frac{d^3k}{(2\pi)^3 v^0} \sum_s [b_v^\dagger(k,s) b_v(k,s) - d_v^\dagger(k,s) d_v(k,s)] \end{aligned} \tag{3.32}$$

It is not difficult to check the following commutation relations

$$\begin{aligned} &[Q_c, b_v^\dagger(k,s)] = b_v^\dagger(k,s), \qquad [Q_c, b_v(k,s)] = -b_v(k,s) \\ &[Q_c, d_v^\dagger(k,s)] = -d_v^\dagger(k,s), \qquad [Q_c, d_v(k,s)] = d_v(k,s) \end{aligned} \tag{3.33}$$

and also $[Q_c, K_v^\mu] = 0$ thus Q_c is time independent and the theory will describe particles of two types, i.e. particles and antiparticles. Since the vacuum has zero charge, we then obtain

$$Q_c |I>= \{ {}^{+|I>, \ I=1,2}_{-|I>, \ I=3,4} \tag{3.34}$$

Another observable is spin which is described by the Pauli-Lubanski operator W_σ constructed from the angular momentum operator $J^{\mu\nu}$. The infinitesimal generator of Lorentz transformation is given by

$$W_\sigma = -\frac{1}{2}\varepsilon_{\sigma\mu\nu\rho}J^{\mu\nu}K^\rho \tag{3.35}$$

Let $n_\mu = \frac{1}{\sqrt{k^2}}(|\overrightarrow{k}|, -\frac{k^0}{|\overrightarrow{k}|}k_i)$ be a normalized space-like four-vector orthogonal to k_μ, we can introduce the helicity operator as

$$J = \frac{W \cdot n}{\sqrt{k^2}} \tag{3.36}$$

Consider now the action of the operator J on the states $|I>$. The operator K^ρ is replaced by its eigenvalue k^ρ. Choosing the third axis along $\overrightarrow{k}$ and noticing the following commutation relations of $J^{\mu\nu}$

$$\begin{aligned}
[J^{\mu\nu}_{v'}, b^\dagger_v(k,s)] &= \delta_{vv'}\int d^3x \bar{Q}_v(x)\not{v}\overleftarrow{L}^{\mu\nu} v^0 u(v,s)e^{-ik\cdot x} \\
[J^{\mu\nu}_{v'}, d^\dagger_v(k,s)] &= -\delta_{vv'}\int d^3x \bar{v}(v,s)e^{-ik\cdot x}v^0 L^{\mu\nu}\not{v}Q_v(x)
\end{aligned} \tag{3.37}$$

and the property $J|0>= 0$, we obtain the following results

$$\begin{aligned}
Jb^\dagger_v(k,s)|0> &= \sum_{s'}\bar{u}(v,s')\frac{\sigma^{12}}{2}u(v,s)b^\dagger_v(k,s')|0> \\
Jd^\dagger_v(k,s)|0> &= -\sum_{s'}\bar{v}(v,s)\frac{\sigma^{12}}{2}v(v,s')d^\dagger_v(k,s')|0>
\end{aligned} \tag{3.38}$$

Using the relation

$$\bar{u}(v,s')\frac{\sigma^{\mu\nu}}{2}u(v,s) = \bar{v}(v,s')\frac{\sigma^{\mu\nu}}{2}v(v,s) = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}v_\rho s^i_\sigma \tau^i_{ss'} \tag{3.39}$$

where τ^i are the Pauli-matrix and s^i_σ are three normalized space-like four-vector orthogonal to v_μ. Choosing $v_\mu = (1,0,0,0)$, we have

$$J|I>= \begin{cases} +\frac{1}{2}|I>, & I=1,4 \\ -\frac{1}{2}|I>, & I=2,3 \end{cases} \tag{3.40}$$

With the results given in eqs.(3.29) and (3.34), we complete the characterization of states: $|1>$ has charge $+1$ and helicity $+\frac{1}{2}$, $|2>$ charge $+1$ and helicity $-\frac{1}{2}$, $|3>$

charge -1 and helicity $+\frac{1}{2}$ and $|4>$ charge -1 and helicity $-\frac{1}{2}$. For convenience we may denote the states as

$$
\begin{aligned}
&|I>=|e,v,k,s>, \qquad (e=\pm, s=\pm\frac{1}{2}, I=1,2,3,4) \\
&|1>=|+,v,k,\frac{1}{2}>, \qquad |2>=|+,v,k,-\frac{1}{2}> \\
&|3>=|-,v,k,\frac{1}{2}>, \qquad |4>=|-,v,k,-\frac{1}{2}>
\end{aligned} \tag{3.41}
$$

It becomes clear for the structure of the Hilbert space of the HQEFT. The full Hilbert space is the Fock space. Consider now multi-particle states. Denote the creation operators by the collective symbol $a_I^\dagger$, with $I = 1,2,3,4$. A basis of Fock space is generated by the states

$$
a_{I_1}^\dagger(1)...a_{I_n}^\dagger(n)|0> \tag{3.42}
$$

From the anticommutation properties of the creation operators, these states will be antisymmetric in the wave function argument $1,...,n$. In particular, they will vanish if two of those coincide.

D. Propagator in HQEFT

From the commutation rules of operators $b_v, b_v^\dagger, d_v$ and $d_v^\dagger$, the anticommutator of two free fields at arbitrary separations reads

$$
\begin{aligned}
&\{Q_v^\alpha(x), \bar{Q}_{v'}^\beta(x')\} \\
&= \delta_{vv'} \int \frac{d^3k}{(2\pi)^3 v^0} \sum_s [e^{-ik\cdot(x-x')} u^\alpha(v,s)\bar{u}^\beta(v,s) + e^{ik(x-x')} v^\alpha(v,s)\bar{v}^\beta(v,s)]
\end{aligned} \tag{3.43}
$$

where α and β are Dirac indices. Using the completeness in eq.(3.23) and writing the phase space measure as

$$
\frac{d^3k}{(2\pi)^3 v^0} = \frac{d^4k}{(2\pi)^4} 2\pi k\cdot v \delta((k\cdot v)^2)\varepsilon(k\cdot v) \tag{3.44}
$$

with $\varepsilon(u) = \frac{u}{|u|}$, we then obtain

$$
\{Q_v^\alpha(x), \bar{Q}_{v'}^\beta(x')\} = \delta_{vv'} \not{v}_{\alpha\beta} i v\cdot\partial_x [i\Delta(x-x')] \tag{3.45}
$$

where

$$\Delta(x-x') = -i\int \frac{d^4k}{(2\pi)^4}\delta((k\cdot v)^2)\varepsilon(k\cdot v)\frac{1}{2}[e^{-ik(x-x')} - e^{ik(x-x')}]$$

If $x^0 = x'^0$, it is reduced to be

$$\{Q_v^\alpha(x), \bar{Q}_{v'}^\beta(x')\}|_{x^0=x'^0} = \delta_{vv'}\not{v}_{\alpha\beta}v_0^{-1}\delta^3(\overrightarrow{x} - \overrightarrow{x}') \tag{3.46}$$

which agrees with eq.(3.25).

The propagator is defined

$$iS_{vv'}(x-x') =< 0|\mathcal{T}Q_v(x)\bar{Q}_{v'}(x')|0 > \tag{3.47}$$

From the definition of the time-ordered product and the anticommutator relation of the effective fields, it is found that the propagator satisfies

$$i\not{v}v\cdot\partial_x S_{vv'}(x-x') = \delta_{vv'}\delta^4(x-x') \tag{3.48}$$

Its solution is easily read

$$\begin{aligned} &< 0|\mathcal{T}Q_v(x)\bar{Q}_{v'}(x')|0 >= iS_{vv'}(x-x') = \delta_{vv'}\int\frac{d^4k}{(2\pi)^4}e^{-ik(x-x')}\frac{i}{\not{v}v\cdot k + i\varepsilon} \\ &\equiv \delta_{vv'}\int\frac{d^4k}{(2\pi)^4}\frac{i}{\not{v}v\cdot k + i\varepsilon}\left(P_+e^{-i\not{v}k(x-x')} + P_-e^{i\not{v}k(x-x')}\right) \end{aligned} \tag{3.49}$$

with $P_\pm = (1\pm\not{v})/2$ the project operators.

E. Discrete Symmetries in HQEFT

The discrete symmetries are the symmetries under the parity transformation, charge conjugation and time reversal, which are well defined in full QCD. Here we shall extend to the HQEFT of LCQCD.

i). Parity

Let $\mathcal{P}$ the unitary operator of parity transformation, from the field point of view $\mathcal{P}$ acting on the field satisfies in the full theory

$$\mathcal{P}Q(x)\mathcal{P}^\dagger = \eta_p\gamma^0 Q(\tilde{x}), \qquad \tilde{x}^\mu = x_\mu, \quad |\eta_p| = 1 \tag{3.50}$$

It is expected that $\mathcal{P}$ acts on the field Q_v to give an analogous form. What we need to show is how the vector v_μ transforms under the $\mathcal{P}$. For that we may notice the fact that

$$\gamma^0 u(\tilde{v}, s) = u(v, s), \qquad \gamma^0 v(\tilde{v}, s) = -v(v, s) \tag{3.51}$$

which indicates

$$\mathcal{P} Q_v(x) \mathcal{P}^\dagger = \eta_p \gamma^0 Q_{\tilde{v}}(\tilde{x}) \tag{3.52}$$

hence

$$\mathcal{P} b_v(k, s) \mathcal{P}^\dagger = \eta_p b_{\tilde{v}}(\tilde{k}, s), \qquad \mathcal{P} d_v(k, s) \mathcal{P}^\dagger = -\eta_p^* d_{\tilde{v}}(\tilde{k}, s) \tag{3.53}$$

Since the $\gamma^0 Q_{\tilde{v}}(\tilde{x})$ satisfies the parity transformed equation of motion

$$i\not{v} \cdot \partial[\gamma^0 Q_{\tilde{v}}(\tilde{x})] = \gamma^0 i \not{\tilde{v}}\tilde{v} \cdot \tilde{\partial}[Q_{\tilde{v}}(\tilde{x})] = 0 \tag{3.54}$$

It is then expected that $\mathcal{P}$ commutes with Hamiltonian H.

ii). Charge Conjugation

In the full theory the charge conjugate operator $\mathcal{C}$ acting on the field has the condition

$$\mathcal{C} Q(x) \mathcal{C}^\dagger = \eta_c C \bar{Q}^T(x), \qquad \mathcal{C} \bar{Q}(x) \mathcal{C}^\dagger = \eta_c^* Q^T(x) C \tag{3.55}$$

Where T denotes the transposition acting on the Dirac indices. C is the combination of γ matrices. It is straightforward to check that this operation is also valid for field Q_v, when

$$\mathcal{C} b_v(k, s) \mathcal{C} = \eta_c d_v(k, s), \qquad \mathcal{C} d_v(k, s) \mathcal{C} = -\eta_c^* b_v(k, s) \tag{3.56}$$

as C is known satisfying

$$\mathcal{C} \bar{v}^T(v, s) = u(v, s), \qquad \mathcal{C} \bar{u}^T(v, s) = v(v, s) \tag{3.57}$$

One can show that $C\bar{Q}_v^T(x)$ also satisfies the charge conjugated equation of motion

$$i\not{v}\cdot\partial[C\bar{Q}_v^T(x)] = [(\bar{Q}_v i\not{v}\cdot\overleftarrow{\partial})C]^T = 0 \tag{3.58}$$

iii). Time Reversal

Provided that the field $Q_v(x)$ satisfies an analogous transformation as $Q(x)$ does under the time reversal with the antiunitary operator $\mathcal{T}$, the transformation of the four-velocity vector should be clear in this case, since classically we already know the meaning of time-reversal invariance is that by reversing the velocities (space component) in what used to be the final configuration, a system retraces its way back to some original configuration, if the fundamental dynamics has such invariance.

We then expect that $\mathcal{T}$ acting on $Q_v(x)$ satisfies

$$\mathcal{T}Q_v(x)\mathcal{T}^\dagger = \eta_t T Q_{\tilde{v}}(-\tilde{x}) \tag{3.59}$$

which requires

$$\mathcal{T}b_v(k,s)\mathcal{T}^\dagger = \eta_t b_{\tilde{v}}(\tilde{k},s), \qquad \mathcal{T}d_v(k,s)\mathcal{T}^\dagger = -\eta_t^* d_{\tilde{v}}(\tilde{k},s) \tag{3.60}$$

and

$$Tu(v,s) = u^*(\tilde{v},s), \qquad Tv(v,s) = v^*(\tilde{v},s) \tag{3.61}$$

with $T = -i\gamma_5 C$

iv). $\mathcal{CPT}$ Operation

Let us denote $\Theta = \mathcal{CPT}$ as the combined anti-unitary operator which satisfies when acting on field $Q_v(x)$

$$\Theta Q_v(x)\Theta^\dagger = i\gamma_0\gamma_5\bar{Q}_v^T(-x), \qquad \Theta\bar{Q}_v(x)\Theta^\dagger = -Q_v^T(-x)i\gamma_0\gamma_5 \tag{3.62}$$

Finally we would like to show that the Lagrangian $\mathcal{L}_{Q,v}(x)$ transforms under Θ as

$$\Theta\mathcal{L}_{Q,v}(x)\Theta^{\dagger} = \mathcal{L}_{Q,v}(-x) \tag{3.63}$$

For the each discrete transformation we have

$$\begin{aligned} \mathcal{T}\mathcal{L}_{Q,v}(x)\mathcal{T}^{\dagger} &= \mathcal{L}_{Q,\tilde{v}}(-x) \\ \mathcal{P}\mathcal{L}_{Q,v}(x)\mathcal{P}^{\dagger} &= \mathcal{L}_{Q,\tilde{v}}(\tilde{x}) \\ \mathcal{C}\mathcal{L}_{Q,v}(x)\mathcal{C}^{\dagger} &= \mathcal{L}_{Q,v}(x) \end{aligned} \tag{3.64}$$

Noticing the velocity Lagrangian super-equivalence, we conclude that the effective theory is invariance under the parity transformation, charge conjugation and time reversal.

IV. BASIC FRAMEWORK OF HQEFT

A. Feynman Rules in HQEFT

The effective Lagrangian describing the interacting system of the gluon field reads

$$\mathcal{L}^{(0)}_{Q,v} = \bar{Q}_v i\not{v} v\cdot DQ_v \equiv \mathcal{L}^{(00)}_{Q,v} + \mathcal{L}^{(I)}_{Q,v} \tag{4.1}$$

where

$$\mathcal{L}^{(00)}_{Q,v} = \bar{Q}_v i\not{v} v\cdot\partial Q_v, \qquad \mathcal{L}^{(I)}_{Q,v} = \bar{Q}_v i\not{v} v^{\mu}(-igA^a_{\mu}T^a)Q_v$$

In the interaction representation and using perturbative theory technology, it is not difficult to obtain the Feynman rules of the HQEFT with analogous procedure as in the full QCD theory. The propagator of heavy quarks is

$$\frac{i}{\not{v} v\cdot k} \tag{4.2}$$

and the gluon-heavy quark vertex

$$ig\not{v} v_{\mu}T^a(2\pi)^4\delta^4(k-k'-q) \tag{4.3}$$

The gluon propagator and interactions among them are the same as in the full QCD theory.

B. Effective Lagrangian of HQEFT with Finite Mass corrections

We now turn to the finite mass case in which the total Lagrangian can be written as

$$\mathcal{L}_{HQEFT} = \mathcal{L}^{(0)}_{Q,v} + \mathcal{L}^{(1/m_Q)}_{Q,v} = \mathcal{L}^{(00)}_{Q,v} + \mathcal{L}^{(I)}_{Q,v} + \mathcal{L}^{(1/m_Q)}_{Q,v} \tag{4.4}$$

where $\mathcal{L}^{(00)}_{Q,v}$ and $\mathcal{L}^{(I)}_{Q,v}$ are given in the previous subsection. $\mathcal{L}^{(1/m_Q)}_{Q,v}$ represents the finite mass corrections and has the form in terms of the expansion of inverse powers of heavy quark mass m_Q

$$\begin{aligned}
\mathcal{L}^{(1/m_Q)}_{Q,v} &= \frac{1}{2m_Q}\bar{Q}_v i\not{D}_\perp\Big(1-\frac{i\not{v}v\cdot D}{2m_Q}\Big)^{-1} i\not{D}_\perp Q_v \\
&+ \frac{1}{2m_Q}\bar{Q}_v(i\overleftarrow{\not{\mathcal{D}}}_v)e^{2i\not{v}m_Q v\cdot x}\Big(1-\frac{i\not{v}v\cdot D}{2m_Q}\Big)^{-1}(i\not{D}_\perp)Q_v \qquad (4.5)\\
&\equiv \frac{1}{2m_Q}\bar{Q}_v i\not{D}_\perp\Big(1-\frac{i\not{v}v\cdot D}{2m_Q}\Big)^{-1} i\not{D}_\perp Q_v \\
&+ \frac{1}{2m_Q}\bar{Q}_v(-i\overleftarrow{\not{D}}_\perp)\Big(1-\frac{-i\not{v}v\cdot \overleftarrow{D}}{2m_Q}\Big)^{-1}e^{-2i\not{v}m_Q v\cdot x}(i\not{\mathcal{D}}_v)Q_v, \qquad (4.6)
\end{aligned}$$

where

$$\begin{aligned}
i\not{\mathcal{D}}_v &= i\not{v}v\cdot D + \frac{1}{2m_Q}i\not{D}_\perp\Big(1-\frac{i\not{v}v\cdot D}{2m_Q}\Big)^{-1} i\not{D}_\perp \\
i\overleftarrow{\not{\mathcal{D}}}_v &= -i\not{v}v\cdot \overleftarrow{D} + \frac{1}{2m_Q}(-i\overleftarrow{\not{D}}_\perp)\Big(1-\frac{-i\not{v}v\cdot \overleftarrow{D}}{2m_Q}\Big)^{-1}(-i\overleftarrow{\not{D}}_\perp). \qquad (4.7)
\end{aligned}$$

Here the finite mass correction term $\mathcal{L}^{(1/m_Q)}_{Q,v}$ has been expressed by two identities eq.(4.5) and eq.(4.6) for convenience of use. Their forms are mainly different in the ordering of operators. The first expression eq.(4.5) is used when the effective heavy quark field $\bar{Q}_v$ becomes a virtual or internal one, and the second expression eq.(4.6) is adopted when the effective heavy quark field Q_v is a virtual or internal one. This is because the virtual or internal effective heavy quark will pick up an additional virtual momentum of $2m_Q v$ due to the factor $e^{\mp 2i\not{v}m_Q v\cdot x}$ which arises from the momentum shift for effective heavy quark and antiquark fields $\hat{Q}^{(\pm)}_v \rightarrow e^{-i\not{v}m_Q v\cdot x}Q^{(\pm)}_v = e^{\mp i m_Q v\cdot x}Q^{(\pm)}_v$. While if one uses the effective Lagrangian given in eq.(2.26), there appears no such an extra momentum factor.

The $\mathcal{L}_v^{(1/m_Q)}$ term should be small in the sense of effective theory and will be therefore treated as a perturbative term in the perturbative theory. We would like to point out that to compute the corrections of $\frac{1}{m_Q}$ to Green functions and therefore to physical observables one has to add contributions arising from the mixing terms of the effective quark and antiquark fields in the effective Lagrangian. In this sense, HQEFT contains no non-local interactions.

C. Trivialization of Gluon Couplings and Decouple Theorem

The gluon couplings in the HQEFT of LCQCD can be trivialized in a similar way as an elimination of the mass term. Such a trivialization has been shown in QED by Bloch and Nordsieck to hold in the limit of going on shell. The trivialization in QCD can be arrived by the change of variable (Wilson-line transformation)[19]

$$\begin{aligned} Q_v &= \mathcal{P} e^{ig\int_{-\infty}^{v\cdot x} d\tau v\cdot A^a T^a} Q_v^0 \equiv W(x,v) Q_v^0 \\ \bar{Q}_v &= \bar{Q}_v^0 \mathcal{P} e^{-ig\int_{-\infty}^{v\cdot x} d\tau v\cdot A^a T^a} \equiv \bar{Q}_v^0 W^{-1}(x,v) \end{aligned} \tag{4.8}$$

where $\mathcal{P}$ denotes path ordering with $x^\mu = v^\mu \tau$. Notice that

$$\begin{aligned} v\cdot D Q_v &= \mathcal{P} e^{ig\int_{-\infty}^{v\cdot x} v\cdot A^a T^a} v\cdot\partial Q_v^0 \\ (\not{D} - \not{v} v\cdot D) Q_v &= \mathcal{P} e^{ig\int_{-\infty}^{v\cdot x} v\cdot A^a T^a} (\not{D} - \not{v} v\cdot D) Q_v^0 \end{aligned} \tag{4.9}$$

In terms of the new effective fields, the HQEFT Lagrangian becomes

$$\mathcal{L}_{Q,v} = \bar{Q}_v^0 i\not{v} v\cdot\partial Q_v^0 + \mathcal{L}_{Q,v}^{(1/m_Q)} \tag{4.10}$$

with

$$\begin{aligned} \mathcal{L}_{Q,v}^{(1/m_Q)} &= \frac{1}{2m_Q}\bar{Q}_v^0 i\not{D}_\perp \Big(1 - \frac{i\ \not{v} v\cdot\partial}{2m_Q}\Big)^{-1} i\not{D}_\perp Q_v^0 \\ &+ \frac{1}{2m_Q}\bar{Q}_v^0 (i\overleftarrow{\not{D}}_v) e^{2i\not{v} m_Q v\cdot x}\Big(1 - \frac{i\ \not{v} v\cdot\partial}{2m_Q}\Big)^{-1} (i\not{D}_\perp) Q_v^0 \qquad (4.11) \\ &\equiv \frac{1}{2m_Q}\bar{Q}_v^0 i\not{D}_\perp \Big(1 - \frac{i\ \not{v} v\cdot\partial}{2m_Q}\Big)^{-1} i\not{D}_\perp Q_v^0 \\ &+ \frac{1}{2m_Q}\bar{Q}_v^0 (-i\overleftarrow{\not{D}}_\perp)\Big(1 - \frac{-i\ \not{v} v\cdot\overleftarrow{\partial}}{2m_Q}\Big)^{-1} e^{-2i\not{v} m_Q v\cdot x} (i\not{D}_v) Q_v^0, \qquad (4.12) \end{aligned}$$

where

$$i\not{\mathcal{D}}_v = i\ \not{v}\cdot\partial + \frac{1}{2m_Q} i\not{D}_\perp \Big(1 - \frac{i\ \not{v}\cdot\partial}{2m_Q}\Big)^{-1} i\not{D}_\perp$$

$$i\overleftarrow{\not{\mathcal{D}}}_v = -i\ \not{v}\cdot\overleftarrow{\partial} + \frac{1}{2m_Q}(-i\overleftarrow{\not{D}}_\perp)\Big(1 - \frac{-i\ \not{v}\cdot\overleftarrow{\partial}}{2m_Q}\Big)^{-1}(-i\overleftarrow{\not{D}}_\perp). \qquad (4.13)$$

It becomes manifest that in the infinite mass limit the new effective field Q_v^0 behaves like a free field, which implies that the heavy quarks decouple from the theory in the infinity mass limit case. In another word the contributions of the heavy quarks to the process are suppressed by orders in $1/m_Q$. This naturally leads to the decouple theorem of heavy quarks in the strong QCD interactions. This statement holds for both perturbative and non-perturbative cases.

If there are N_f flavors of heavy quarks in a theory, transformations indicated in eq.(4.8) with appropriate velocity dependent paths will lead the effective Lagrangian to a similar expression except labelled by their own velocity. Note that if there is only one heavy flavor in a theory, the gluon field can be simply eliminated by fixing the gauge to be $v \cdot A = 0$. In the case of N_f flavors of heavy quarks, it can be realized only in the case that all quarks move at the same velocity. Otherwise it is not possible to choose a gauge to eliminate such gluon couplings for all sectors of flavors.

D. Renormalization in HQEFT and Wilson Loops

Let us now discuss the renormalizability of the HQEFT. To the leading order in $1/m_Q$, the HQEFT in terms of the effective field Q_v is already power counting renormalizable. Nevertheless its renormalizability will be more manifest in terms of the effective field Q_v^0 and Wilson lines. This is because the Wilson lines have been proved to be renormalizable and the effective field Q_v^0 is a free one that suffer no dressing to all order of QCD. Note that when including the terms in $1/m_Q$, i.e. $\mathcal{L}_v^{(1/m_Q)}$, which is not renormalizable from the power counting, it is difficult to make precise evaluations of $1/m_Q$ effects. However, $\mathcal{L}_v^{(1/m_Q)}$ is suppressed by the powers of $1/m_Q$ in the sense of effective field theory, it will be treated as a simple insertion in Green functions in the perturbative expansion.

To illustrate the renormalization in the HQEFT of LCQCD, we consider, as a simple example, the renormalization of the operator

$$J^{(0)}_{Q,v} = \bar{Q}'_{v'}(x)\Gamma Q_v(x) \tag{4.14}$$

In terms of the new variable Q^0_v, we have

$$\begin{aligned} J^{(0)}_{Q,v} &= \bar{Q}'^0_{v'}(x)\mathcal{P}e^{-ig\int_{-\infty}^{v'\cdot x} v'\cdot A^a T^a}\Gamma\ \mathcal{P}e^{-ig\int_{-\infty}^{v\cdot x} v\cdot A^a T^a} Q^0_v \\ &\equiv \bar{Q}'^0_{v'} W^{-1}(x,v')W(x,v)\Gamma Q^0_v \end{aligned} \tag{4.15}$$

As shown from above subsection that to the leading order in $1/m_Q$ the effective field $Q^0_v(x)$ is with respect to the free field and therefore suffers no renormalization to all orders in the coupling. The renormalization of the operator J^{eff}_Γ is then reduced to the renormalization of the Wilson loops, i.e.

$$\frac{1}{N_c} Tr < 0|W^{-1}(x,v')W(x,v)|0 >= \frac{1}{N_c} < 0|\mathcal{P}e^{ig\oint_{C_\delta} dx^\mu A_\mu}|0 >\equiv W(C_\delta) \tag{4.16}$$

where C_δ is the loop with, at the point $v'\cdot x = v\cdot x$, a cusp characterized by angle δ. In Minkowski space the angle δ is given by

$$\cosh\delta = v\cdot v' \tag{4.17}$$

It was shown that a Wilson loop is multiplicatively renormalizable in the case of having a finite number of self-interaction points and cusps corresponding to angles $\{\delta_i\}$. The renormalization properties of the Wilson loops containing cusp singularities have been studied in ref.[20, 21]. The two-loop contribution to the cusp anomalous dimension $\Gamma_{cusp}(\delta, g)$ was calculated in [21]. It was found that the renormalized contour average W_R of Wilson loop in the presence of a cusp can be constructed by applying the ordinary $\mathcal{R}$-operation.

$$A^\mu \to A^\mu_R = Z_3^{-\frac{1}{2}} A^\mu, \qquad g\to g_R = Z_1^{-1} Z_3^{\frac{3}{2}} \mu^{\frac{\varepsilon}{2}} g \tag{4.18}$$

with incorporating the subtraction procedure $\mathcal{K}_\delta$

$$W_R(C_\delta; g_R, \mu, \bar{C}_\delta) = \lim_{\varepsilon\to 0}\mathcal{K}_\delta \tilde{W}(C_\delta; g_R, \mu, \varepsilon) = \lim_{\varepsilon\to 0}\mathcal{K}_\delta\mathcal{R}W(C_\delta; g, \varepsilon) \tag{4.19}$$

where $\bar{C}_\delta$ denotes a generalized subtraction point of the $\mathcal{K}_\delta$ procedure. It is also proved that the renormalized contour average $W_R(C_\delta; g_R, \mu, \bar{C}_\delta)$ satisfies the exponential theorem

$$W_R(C_\delta; g_R, \mu, \bar{C}_\delta) = e^{W_R^{2PI}(C_\delta; g_R, \mu, \bar{C}_\delta)} \tag{4.20}$$

where W_R^{2PI} is the two-particle irreducible contour averages, and satisfies the renormalization group equation

$$[\mu\frac{\partial}{\partial\mu} + \beta(g_R)\frac{\partial}{\partial g_R}]W_R(C_\delta; g_R, \mu, \bar{C}_\delta) = -\Gamma_{cusp}(\delta, g_R) \tag{4.21}$$

$\Gamma_{cusp}(\delta, g_R)$ is the anomalous dimension defined by

$$\Gamma_{cusp}(\delta, g_R) = -\lim_{\varepsilon\to 0}\frac{d}{d(\ln\mu)}\tilde{W}^{2PI}(C_\delta; g_R, \mu, \varepsilon) \tag{4.22}$$

The general solution of the renormalization group equation can be written

$$W_R(C_\delta; g_R, \mu, \bar{C}_\delta) = W_R(C_\delta; g_R, \bar{\mu}, \bar{C}_\delta)e^{-\int_{g_R(\bar{\mu})}^{g_R(\mu)} dg \frac{\Gamma_{cusp}(\delta, g)}{\beta(g)}} \tag{4.23}$$

Note that the cusp anomalous dimension $\Gamma_{cusp}(\delta, g)$ depends only on the cusp angle δ. One- and two-loop contributions to the cusp anomalous dimension have been calculated explicitly in ref.[21]. To be manifest, let us quote , for example , the one-loop result

$$\Gamma_{cusp}^{one-loop}(\delta, g_R) = \frac{\alpha_s}{\pi}C_F(\delta\coth\delta - 1) \tag{4.24}$$

which was also reproduced from directly calculating the one-loop QCD corrections of heavy quark currents[22].

E. Current Operators in HQEFT

Just like the effective Lagrangian is established from the full theory, the current operators in the full QCD theory may be expressed in terms of the new effective fields in HQEFT of LCQCD. As an example, consider the current operator

$$J_Q = \bar{Q}'(x)\Gamma Q(x) \tag{4.25}$$

which relates to the current operators in the HQEFT of LCQCD via a sequence of field redefinitions given in the previous sections.

$$
\begin{aligned}
J_Q \to \quad & J_{Q,v} = \bar{Q}'_{v'}(x) e^{i\not{v}' m_{Q'} v'\cdot x} \Gamma e^{-i\not{v} m_Q v\cdot x} Q_v(x) \\
& + \bar{Q}'_{v'}(x) e^{i\not{v}' m_{Q'} v'\cdot x} \Gamma e^{i\not{v} m_Q v\cdot x} W_v(\frac{1}{m_Q}) Q_v(x) \\
& + \bar{Q}'_{v'}(x) \overleftarrow{W}_{v'}(\frac{1}{m_{Q'}}) e^{-i\not{v}' m_{Q'} v'\cdot x} \Gamma e^{-i\not{v} m_Q v\cdot x} Q_v(x) \\
& + \bar{Q}'_{v'}(x) \overleftarrow{W}_{v'}(\frac{1}{m_{Q'}}) e^{-i\not{v}' m_{Q'} v'\cdot x} \Gamma e^{i\not{v} m_Q v\cdot x} W_v(\frac{1}{m_Q}) Q_v(x) \qquad (4.26)
\end{aligned}
$$

Noticing the following identity

$$
e^{\epsilon' i\not{v}' m_{Q'} v'\cdot x} \Gamma e^{-\epsilon i\not{v} m_Q v\cdot x} \equiv \sum_{\sigma'=\pm} \sum_{\sigma=\pm} \frac{1+\epsilon'\sigma'\not{v}'}{2} \Gamma \frac{1+\epsilon\sigma\not{v}}{2} e^{\epsilon'\sigma' i m_{Q'} v'\cdot x - \epsilon\sigma i m_Q v\cdot x} \qquad (4.27)
$$

with $\epsilon' = \pm,\ \epsilon = \pm$. Then eq.(4.26) can be rewritten as

$$
\begin{aligned}
J_{Q,v} = \quad & \sum_{\sigma'=\pm} \sum_{\sigma=\pm} e^{\sigma' i m_{Q'} v'\cdot x - \sigma i m_Q v\cdot x} [\bar{Q}'_{v'}(x) \frac{1+\sigma'\not{v}'}{2} \Gamma \frac{1+\sigma\not{v}}{2} Q_v(x) \\
& + \bar{Q}'_{v'}(x) \frac{1+\sigma'\not{v}'}{2} \Gamma \frac{1+\sigma\not{v}}{2} W_v(\frac{1}{m_Q}) Q_v(x) \\
& + \bar{Q}'_{v'}(x) \overleftarrow{W}_{v'}(\frac{1}{m_{Q'}}) \frac{1+\sigma'\not{v}'}{2} \Gamma \frac{1+\sigma\not{v}}{2} Q_v(x) \\
& + \bar{Q}'_{v'}(x) \overleftarrow{W}_{v'}(\frac{1}{m_{Q'}}) \frac{1+\sigma'\not{v}'}{2} \Gamma \frac{1+\sigma\not{v}}{2} W_v(\frac{1}{m_Q}) Q_v(x)] \\
\equiv \quad & J^{(0)}_{Q,v} + J^{(1/m_Q)}_{Q,v} \qquad (4.28)
\end{aligned}
$$

where the summation contains four cases which correspond to the transitions between quarks and between antiquarks as well as the creation and annihilation of the quark and antiquark pairs.

Beyond the tree level, the above expression has to be replaced by a more general sum over operators. Note that in computing the coefficient functions of operators to the order in $1/m_Q$ one should include the graphs with an insertion of the terms in the same order of $1/m_Q$ from the effective Lagrangian. In particular, we would like to stress that in the computation of the coefficient functions of the terms of order in $1/m_Q$, there is also a contribution from the terms of the quark-antiquark coupling terms in the effective Lagrangian. For instance, integrating over the antiquark field,

the resulting effective current for quark field receive contributions starting from the order of $1/m_Q$. In general, the effective current can be written as follows after integrating over the antiquark field

$$J^{(++)}_{eff} = J^{(0)}_{eff} + J^{(1/m_Q)}_{eff} \tag{4.29}$$

with

$$J^{(0)}_{eff} = e^{i(m_{Q'}v'-m_Q v)\cdot x}\bar{Q}^{(+)}_{v'}\Gamma Q^{(+)}_v \tag{4.30}$$

$$\begin{aligned}J^{(1/m_Q)}_{eff} &= e^{i(m_{Q'}v'-m_Q v)\cdot x}\{\frac{1}{2m_Q}\bar{Q}^{(+)}_{v'}\Gamma\frac{1}{i\,\not v\cdot D}(i\not D_\perp)^2 Q^{(+)}_v \\ &+\frac{1}{2m_{Q'}}\bar{Q}^{(+)}_{v'}(-i\,\overleftarrow{\not D}_\perp)^2\frac{1}{-i\not v\cdot\overleftarrow{D}}\Gamma Q^{(+)}_v+\frac{1}{4m_Q^2}\bar{Q}^{(+)}_{v'}\Gamma\frac{1}{i\,\not v\cdot D}i\not D_\perp(i\,\not v\cdot D) \\ &\times i\not D_\perp Q^{(+)}_v+\frac{1}{4m_{Q'}^2}\bar{Q}^{(+)}_{v'}(-i\,\overleftarrow{\not D}_\perp)(-i\not v\cdot\overleftarrow{D})(-i\,\overleftarrow{\not D}_\perp)\frac{1}{-i\not v\cdot\overleftarrow{D}}\Gamma Q^{(+)}_v \\ &+\frac{1}{4m_{Q'}m_Q}\bar{Q}^{(+)}_{v'}(-i\,\overleftarrow{\not D}_\perp)^2\times\frac{1}{-i\not v\cdot\overleftarrow{D}}\Gamma\frac{1}{i\,\not v\cdot D}(i\not D_\perp)^2Q^{(+)}_v+O(\frac{1}{m^3_{Q^{(\prime)}}})\}\end{aligned} \tag{4.31}$$

where we only keep to the order of $1/m_Q^2$. We have also used the operator condition $(i\not D_\perp)^2/2m_Q \ll i\,\not v\cdot D$ in the heavy quark expansion, which holds for the typical case with $\langle iD_\parallel\rangle \sim \langle iD_\perp\rangle \sim \bar\Lambda \ll m_Q$ in the heavy-light hadron system.

We would like to address that the above forms of effective current are quite different from those without considering the contributions of antiquark fields. Especially, the terms $\frac{1}{2m_Q}\bar{Q}^{(+)}_{v'}\Gamma i\not D_\perp Q^{(+)}_v$ and $\frac{1}{2m_{Q'}}\bar{Q}^{(+)}_{v'}(-i\,\overleftarrow{\not D}_\perp)\Gamma Q^{(+)}_v$ disappear in HQEFT. This is because they are exactly cancelled by the additional contributions arising from the intermediate antiquark fields. As a consequence, the operator forms in the above effective current $J^{(++)}_{eff}$ and those in the effective Lagrangian $L^{(++)}_{eff}$ become similar, namely all the odd powers of the transverse momentum operator $\not D_\perp$ are absent, only the even powers of $\not D_\perp$ appear in the effective current $J^{(++)}_{eff}$ and effective Lagrangian $L^{(++)}_{eff}$. Such an interesting feature in HQEFT becomes remarkable in evaluating the hadronic matrix elements. For instance, fewer form factors are involved, and $1/m_Q$ corrections at zero recoil are automatically absent for both transitions between heavy pseudoscalar to heavy vector and between heavy pseudoscalar to heavy pseudoscalar mesons.

F. Spin and Angular Momentum in HQEFT

To discuss the spin and rotational symmetries in HQEFT of LCQCD, it is useful to construct the generators by introducing the following Pauli-Lubanski vector

$$W_\sigma = -\frac{1}{2}\varepsilon_{\sigma\mu\nu\rho} J^{\mu\nu} v^\rho \tag{4.32}$$

where $J^{\mu\nu}$ is the angular momentum operator, i.e. the infinitesimal generator of Lorentz transformation.

$$J^{\mu\nu} = \frac{1}{2}\sigma^{\mu\nu} + i(x^\mu \partial^\nu - x^\nu \partial^\mu) \tag{4.33}$$

Substituting (4.33) into (4.32), we obtain

$$W_\sigma = S_\sigma + L_\sigma \tag{4.34}$$

with

$$\begin{aligned} S_\sigma &= -\frac{1}{4}\varepsilon_{\sigma\mu\nu\rho}\sigma^{\mu\nu} v^\rho = \frac{i}{2}\gamma_5 \sigma_{\sigma\rho} v^\rho = -\frac{1}{2}\gamma_5 (\gamma_\sigma - v_\sigma \not{v})\not{v} \\ L_\sigma &= -\frac{i}{2}\varepsilon_{\sigma\mu\nu\rho}(x^\mu \partial^\nu - x^\nu \partial^\mu) v^\rho \end{aligned} \tag{4.35}$$

It is not difficult to show that the effective Lagrangian in the infinite mass limit is invariant under the transformations

$$Q_v \to e^{iS_\sigma \alpha^\sigma}, \qquad \bar{Q}_v \to \bar{Q}_v e^{-iS_\sigma \alpha^\sigma} \tag{4.36}$$

$$Q_v \to e^{iL_\sigma \alpha^\sigma}, \qquad \bar{Q}_v \to \bar{Q}_v e^{-iL_\sigma \alpha^\sigma} \tag{4.37}$$

and therefore

$$Q_v \to e^{iW_\sigma \alpha^\sigma}, \qquad \bar{Q}_v \to \bar{Q}_v e^{-iW_\sigma \alpha^\sigma} \tag{4.38}$$

At the rest frame with $v = (1, 0, 0, 0)$, the angular momentum can be written as

$$W_i = L_i + S_i \tag{4.39}$$

with

$$\begin{aligned} S_i &= \frac{1}{4}\varepsilon_{ijk}\sigma^{jk} = \frac{1}{2}\gamma^5\gamma^0\gamma^i = \frac{1}{2}\begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix} \\ L_i &= \frac{i}{2}\varepsilon_{ijk}(x^j \partial^k - x^k \partial^j) \end{aligned} \tag{4.40}$$

where S_i are the usual spin-matrices and L_i are the usual orbital angular momentum operators. This shows that the spin $\overrightarrow{S}$ and orbital angular momentum $\overrightarrow{L}$ are separately conserved, angular momentum $\overrightarrow{W}$ is therefore also conserved in infinite mass limit.

G. New Symmetries in HQEFT at infinite mass limit

In addition to spin-flavor symmetry, we shall show that the effective Lagrangian in infinite mass limit is also invariant under the following transformations

$$Q_v \to e^{i(\alpha_5\gamma_5+\alpha)}Q_v, \qquad \bar{Q}_v \to \bar{Q}_v e^{i(\alpha_5\gamma_5-\alpha)} \tag{4.41}$$

$$Q_v \to e^{i\not{v}(ia_5\gamma_5+a)}Q_v, \qquad \bar{Q}_v \to \bar{Q}_v e^{i\not{v}(a_5\gamma_5-a)} \tag{4.42}$$

$$Q_v \to e^{i(\gamma_\mu - v_\mu\not{v})(b_5^\mu\gamma_5+ib^\mu)}Q_v, \qquad \bar{Q}_v \to \bar{Q}_v e^{-i(\gamma_\mu - v_\mu\not{v})(b_5^\mu\gamma_5-ib^\mu)} \tag{4.43}$$

$$\begin{aligned} Q_v &\to e^{i(v_\mu\gamma_\nu - v_\nu\gamma_\mu)(c_5^{\mu\nu}\gamma_5+ic^{\mu\nu})}Q_v \\ \bar{Q}_v &\to \bar{Q}_v e^{-i(v_\mu\gamma_\nu - v_\nu\gamma_\mu)(c_5^{\mu\nu}\gamma_5-ic^{\mu\nu})} \end{aligned} \tag{4.44}$$

$$\begin{aligned} Q_v &\to e^{i[\sigma_{\mu\nu}+i(v_\mu\gamma_\nu - v_\mu\gamma_\nu)\not{v}](d_5^{\mu\nu}\gamma_5+d^{\mu\nu})}Q_v \\ \bar{Q}_v &\to \bar{Q}_v e^{i[\sigma_{\mu\nu}+i(v_\mu\gamma_\nu - v_\nu\gamma_\mu)\not{v}](d_5^{\mu\nu}\gamma_5-d^{\mu\nu})} \end{aligned} \tag{4.45}$$

$$Q_v \to e^{i\not{v}\not{\varepsilon}(f_5\gamma_5+f)}Q_v, \qquad \bar{Q}_v \to \bar{Q}_v e^{-i\not{v}\not{\varepsilon}(f_5\gamma_5-f)} \tag{4.46}$$

$$Q_v \to e^{i\gamma_5\not{\varepsilon}g}Q_v, \qquad \bar{Q}_v \to \bar{Q}_v e^{-i\gamma_5\not{\varepsilon}g} \tag{4.47}$$

where ε_μ is the polarization vector with $\varepsilon \cdot v = 0$.

V. HADRONIC MATRIX ELEMENTS AND $1/m_Q$ EXPANSIONS IN HQEFT

As a direct application of HQEFT, we are going to show how the hadronic matrix element in full QCD theory can systematically be expanded into a series of matrix elements in terms of $1/m_Q$ in HQEFT.

A. Weak Transition Matrix Elements in HQEFT

Adopting the conventional relativistic normalization

$$< H(p')|H(p) >= 2p^0(2\pi)^3\delta^3(\vec{p}\,' - \vec{p}), \tag{5.1}$$

the conservation of the vector current $\bar{Q}\gamma^\mu Q$ leads to

$$< H(p)|\bar{Q}\gamma^\mu Q|H(p) >= 2p^\mu = 2m_H v^\mu, \tag{5.2}$$

where $|H >$ denotes a hadron state in QCD and $p^\mu = m_H v^\mu$ is the momentum of the heavy hadron H. This may be regarded as an alternative definition of heavy hadron mass.

To exhibit a manifest spin-flavor symmetry in HQEFT at infinite mass limit, we introduce an effective heavy hadron state $|H_v >$ with the normalization

$$< H_v|\bar{Q}_v\gamma^\mu Q_v|H_v >= 2\bar{\Lambda}v^\mu \tag{5.3}$$

where

$$\bar{\Lambda} = \lim_{m_Q\to\infty} \bar{\Lambda}_H$$

is a heavy flavor-independent binding energy that reflects the confinement effects of the light degrees of freedom in the heavy hadron.

The hadronic matrix element in full QCD is then given by the one in HQEFT via the following formulation

$$\frac{1}{\sqrt{m_{H'}m_H}} < H'|J_Q|H >= \frac{1}{\sqrt{\bar{\Lambda}_{H'}\bar{\Lambda}_H}} < H'_{v'}|J_{Q,v}e^{i\int d^4x\mathcal{L}_{HQEFT}}|H_v > . \tag{5.4}$$

Here $\bar{\Lambda}_H$ and $\bar{\Lambda}_{H'}$ are the binding energies defined as

$$\bar{\Lambda}_H \equiv m_H - m_Q, \qquad \bar{\Lambda}_{H'} \equiv m_{H'} - m_{Q'}. \tag{5.5}$$

The flavor-dependent factor $\frac{1}{\sqrt{\bar{\Lambda}_{H'}\bar{\Lambda}_H}}$ appears due to the different normalization of the hadron states $|H >$ in full QCD and $|H_v >$ in HQEFT.

In general, the $1/m_Q$ corrections to the hadronic matrix elements in the heavy quark expansion can be classified into three parts: 1) corrections from purely effective current $J_{Q,v}^{(1/m_Q)}$; 2) corrections from purely effective Lagrangian $\mathcal{L}_{Q,v}^{(1/m_Q)}$; and 3) mixed corrections from both $J_{Q,v}^{(1/m_Q)}$ and $L_{Q,v}^{(1/m_Q)}$.

After a detailed evaluation with contracting the effective heavy quark field, the hadronic matrix element in HQEFT is found to have the following general form up to order of $1/m_Q^2$

$$
\begin{aligned}
\mathcal{A} &\equiv \; < H'_{v'}|J_{Q,v} e^{i\int d^4x \mathcal{L}_{HQEFT}}|H_v> \\
&= \; < H'_{v'}|\bar{Q}'_{v'}\Gamma Q_v|H_v> - \frac{1}{2m_Q} < H'_{v'}|\bar{Q}'_{v'}O_1(\Gamma)Q_v|H_v> \\
& \quad - \frac{1}{2m_{Q'}} < H'_{v'}|\bar{Q}'_{v'}O'_1(\Gamma)Q_v|H_v> - \frac{1}{4m_Q^2} < H'_{v'}|\bar{Q}'_{v'}O_2(\Gamma)Q_v|H_v> \\
& \quad - \frac{1}{4m_{Q'}^2} < H'_{v'}|\bar{Q}'_{v'}O'_2(\Gamma)Q_v|H_v> - \frac{1}{4m_Q^2} < H'_{v'}|\bar{Q}'_{v'}O_3(\Gamma)Q_v|H_v> \\
& \quad - \frac{1}{4m_{Q'}^2} < H'_{v'}|\bar{Q}'_{v'}O'_3(\Gamma)Q_v|H_v> + \frac{1}{4m_{Q'}m_Q} < H'_{v'}|\bar{Q}'_{v'}O_4(\Gamma)Q_v|H_v> \\
& \quad + O(\frac{1}{m_{Q'}^3}). \qquad (5.6)
\end{aligned}
$$

which explicitly displays the $1/m_Q$ expansion. Where the operators $O_i(\Gamma)$ and $O'_i(\Gamma)$ are defined as follows

$$
\begin{aligned}
O_1(\Gamma) &= \Gamma \frac{1}{i \not{v}\cdot\partial}(i\not{D}_\perp)^2, \\
O'_1(\Gamma) &= (-i \overleftarrow{\not{D}}_\perp)^2 \frac{1}{-i\not{v}\cdot\overleftarrow{\partial}}\Gamma, \\
O_2(\Gamma) &= \Gamma \frac{1}{i \not{v}\cdot\partial}(i\not{D}_\perp)(i\not{v}\cdot D)i\not{D}_\perp, \\
O'_2(\Gamma) &= (-i \overleftarrow{\not{D}}_\perp)(-i\not{v}\cdot\overleftarrow{D})(-i \overleftarrow{\not{D}}_\perp)\frac{1}{-i\not{v}\cdot\overleftarrow{\partial}}\Gamma, \\
O_3(\Gamma) &= \Gamma \frac{1}{i \not{v}\cdot\partial}(i\not{D}_\perp)^2 \frac{1}{i \not{v}\cdot\partial}(i\not{D}_\perp)^2, \\
O'_3(\Gamma) &= (-i \overleftarrow{\not{D}}_\perp)^2 \frac{1}{-i\not{v}\cdot\overleftarrow{\partial}}(-i \overleftarrow{\not{D}}_\perp)^2 \frac{1}{-i\not{v}\cdot\overleftarrow{\partial}}\Gamma, \\
O_4(\Gamma) &= (-i \overleftarrow{\not{D}}_\perp)^2 \frac{1}{-i\not{v}\cdot\overleftarrow{\partial}}\Gamma \frac{1}{i \not{v}\cdot D}(i\not{D}_\perp)^2. \qquad (5.7)
\end{aligned}
$$

which are all given in the even powers of $\not{D}_\perp$. Note that the term $\frac{1}{i\not{v}\cdot\partial}$ (or $\frac{1}{i\not{v}\cdot\overleftarrow{\partial}}$) arises with replacing the propagator from contracting effective heavy quark fields in HQEFT by the corresponding operator.

B. Mass Formula of Hadrons and Transition Matrix Elements at Zero Recoil in HQEFT

Based on the formulation eq.(5.4) and the normalization conditions for hadron states in full QCD (eq.(5.2)) and in HQEFT (eq.(5.3)), by setting $v' = v$, we obtain

$$\begin{aligned} 2m_H v^\mu &= < H|Q\gamma^\mu Q|H >= \frac{m_H}{\bar{\Lambda}_H}\{2\bar{\Lambda}v^\mu - \frac{1}{m_Q} < H_v|\bar{Q}_v O_1(\gamma^\mu)Q_v|H_v > \\ &- \frac{1}{2m_Q^2} < H_v|\bar{Q}_v(O_2(\gamma^\mu) + O_3(\gamma^\mu))Q_v|H_v > \\ &+ \frac{1}{4m_Q^2} < H_v|\bar{Q}_v O_4(\gamma^\mu)Q_v|H_v > + O(\frac{1}{m_Q^3})\}, \end{aligned} \tag{5.8}$$

which allows us to extract the binding energy in terms of $1/m_Q$ expansion

$$\begin{aligned} \bar{\Lambda}_H &= \bar{\Lambda} - \frac{1}{2m_Q} < H_v|\bar{Q}_v O_1(\not{v})Q_v|H_v > - \frac{1}{4m_Q^2} < H_v|\bar{Q}_v(O_2(\not{v}) + O_3(\not{v})Q_v|H_v > \\ &+ \frac{1}{8m_Q^2} < H_v|\bar{Q}_v O_4(\not{v})Q_v|H_v > + O(\frac{1}{m_Q^3}). \end{aligned} \tag{5.9}$$

We then arrive at an alternative definition for the heavy hadron mass

$$m_H = m_Q + \bar{\Lambda}_H = m_Q + \bar{\Lambda} + O(1/m_Q), \tag{5.10}$$

which shows that the mass of a hadron is given by three parts: the effective heavy quark mass m_Q, the binding energy $\bar{\Lambda}$ due to light degrees of freedom and terms suppressed by $1/m_Q$.

We now turn to discuss the transition matrix elements. For illustration, consider first a simple case that both the initial and final states are pseudoscalar mesons. We may choose a realistic process, i.e., $B \to D$ transition matrix element for vector current. From the formulation eq.(5.4) and the transition matrix elements in the heavy quark expansion eq. (5.6), it is easily read

$$\begin{aligned} < D|\bar{c}\gamma^\mu b|B &>= \sqrt{\frac{m_D m_B}{\bar{\Lambda}_D \bar{\Lambda}_B}}\{< D_{v'}|\bar{c}_{v'}^+ \gamma^\mu b_v^+|B_v > - \frac{1}{2m_b} < D_{v'}|\bar{c}_{v'}^+ O_1(\gamma^\mu)b_v^+|B_v > \\ &- \frac{1}{2m_c} < D_{v'}|\bar{c}_{v'}^+ O_1'(\gamma^\mu)b_v^+|B_v > - \frac{1}{4m_b^2} < D_{v'}|\bar{c}_{v'}^+ O_2(\gamma^\mu)b_v^+|B_v > \\ &- \frac{1}{4m_c^2} < D_{v'}|\bar{c}_{v'}^+ O_2'(\gamma^\mu)b_v^+|B_v > - \frac{1}{4m_b^2} < D_{v'}|\bar{c}_{v'}^+ O_3(\gamma^\mu)b_v^+|B_v > \end{aligned}$$

$$-\frac{1}{4m_c^2} < D_{v'}|\bar{c}_{v'}^{+}O_3'(\gamma^\mu)b_v^{+}|B_v> +\frac{1}{4m_c m_b} < D_{v'}|\bar{c}_{v'}^{+}O_4(\gamma^\mu)b_v^{+}|B_v>$$
$$+O(\frac{1}{m_{b(c)}^3})\}. \tag{5.11}$$

Applying spin-flavor symmetry, we have the following relations for operators O_i

$$< B_v|\bar{b}_v^{+}O_i b_v^{+}|B_v> = < D_v|\bar{c}_v^{+}O_i c_v^{+}|D_v> \equiv < P_v|\bar{Q}_v^{(+)}O_i Q_v^{(+)}|P_v> . \tag{5.12}$$

With the above relations and the binding energy relations eq.(5.9), it is not difficult to find that the $B \to D$ transition matrix element is simplified to the following form at zero recoil $v' = v$,

$$\begin{aligned} < D|\bar{c}\gamma^\mu b|B>|_{q^2_{max}} &= 2\sqrt{m_B m_D}v^\mu\{1+\frac{1}{32\bar{\Lambda}^2}(\frac{1}{m_b}-\frac{1}{m_c})^2 < P_v|\bar{Q}_v^{(+)}O_1(\not{v})Q_v^{(+)}|P_v> \\ &\quad -\frac{1}{16\bar{\Lambda}}(\frac{1}{m_b}-\frac{1}{m_c})^2 < P_v|\bar{Q}_v^{(+)}O_4(\not{v})Q_v^{(+)}|P_v> +O(\frac{1}{m_{b(c)}^3})\}, \end{aligned} \tag{5.13}$$

which explicitly shows that when applying the formulation eq.(5.4) and the normalizations eq.(5.2) and eq.(5.3), the transition matrix elements of heavy quark vector current between two pseudoscalar mesons automatically do not receive corrections of order $1/m_Q$ at zero recoil even without analyzing the concrete Lorentz structure and evaluating the hadronic matrix elements for each operator.

C. Trace Formula of Transition Matrix Elements and Universal Isgur-Wise Function

Evaluating the transition matrix elements is a hard task. The transition matrix element for heavy-light hadrons is generally defined as

$$T_\Gamma = < H'(p')|\int_z \bar{Q}'(z)\Gamma Q(z)e^{iq\cdot z}|H(p)> \tag{5.14}$$

For illustration, we first consider its leading term in HQEFT, which is simply given by

$$\begin{aligned} T_\Gamma^0 &= < H_{v'}'|\int_z \bar{Q}_{v'}'(z)e^{i\not{v}' m_{Q'}v'\cdot z}\Gamma e^{i\not{v} m_Q v\cdot z}Q_v(z)e^{iq\cdot z}|H_v> \\ &= < H_{v'}'|\int_z \bar{Q}_{v'}'(z)\frac{1+\not{v}'}{2}\Gamma\frac{1+\not{v}}{2}Q_v(z)e^{i(q-m_Q v+m_{Q'}v')\cdot z}|H_v> \end{aligned}$$

$$
\begin{aligned}
&+ \; < H'_{v'}| \int_z \bar{Q}'_{v'}(z) \frac{1-\not{v}'}{2} \Gamma \frac{1-\not{v}}{2} Q_v(z) e^{i(q+m_Q v - m_{Q'}v')\cdot z} |H_v > \\
&+ \; < H'_{v'}| \int_z \bar{Q}'_{v'}(z) \frac{1-\not{v}'}{2} \Gamma \frac{1+\not{v}}{2} Q_v(z) e^{i(q-m_Q v - m_{Q'}v')\cdot z} |H_v > \\
&+ \; < H'_{v'}| \int_z \bar{Q}'_{v'}(z) \frac{1+\not{v}'}{2} \Gamma \frac{1-\not{v}}{2} Q_v(z) e^{i(q+m_Q v + m_{Q'}v')\cdot z} |H_v > \qquad (5.15)
\end{aligned}
$$

For the meson state $H_v \equiv M_v = (\bar{Q}_v q)$ with $k = \bar{\Lambda} v$ in HQEFT, the meson state $|H_v >$ can generally be expressed as

$$
\begin{aligned}
|H_v > \; &= \int_k \int_{\tilde{k}} \delta(\bar{\Lambda}_H v - k - \tilde{k}) b_v^+(k,s) d_q^+(\tilde{k},\tilde{s}) \phi_{s\tilde{s}}(k,\tilde{k}) |0 > \\
&\equiv \int_{\tilde{k}} b_v^+(\bar{\Lambda}_H v - \tilde{k}, s) d_q^+(\tilde{k},\tilde{s}) \phi_{s\tilde{s}}(v\cdot\tilde{k}, \tilde{k}^2) |0 > \\
&\equiv \int_k b_v^+(k,s) d_q^+(\bar{\Lambda}_H v - k, \tilde{s}) \phi_{s\tilde{s}}(v\cdot k, k^2) |0 > \qquad (5.16)
\end{aligned}
$$

here we have used the definitions

$$
\begin{aligned}
&\phi_{s\tilde{s}}(\bar{\Lambda}_H v - \tilde{k}, \tilde{k}) \equiv \phi_{s\tilde{s}}(v\cdot\tilde{k}, \tilde{k}^2) \\
&\phi_{s\tilde{s}}(k, \bar{\Lambda}_H v - k) \equiv \phi_{s\tilde{s}}(v\cdot k, k^2) \qquad (5.17)
\end{aligned}
$$

where $\phi_{s\tilde{s}}(k,\tilde{k})$ is the wave function in momentum space. In this case, only the first term in the transition matrix element T^0_Γ becomes non-vanishing

$$
\begin{aligned}
T^0_\Gamma \; &=< H'_{v'}| \int_z \bar{Q}'_{v'}(z) \frac{1+\not{v}'}{2} \Gamma \frac{1+\not{v}}{2} Q_v(z) e^{i(q-m_Q v + m_{Q'}v')\cdot z} |H_v > \\
&=< 0| \int_{k'} \int_{\tilde{k}'} \phi'^{\dagger}_{s'\tilde{s}'}(k',\tilde{k}') b_{v'}(k',s') d_q(\tilde{k}',\tilde{s}') \delta(\bar{\Lambda}_{H'} v' - k' - \tilde{k}') \\
&\int_z \bar{Q}'_{v'}(z) \frac{1+\not{v}'}{2} \Gamma \frac{1+\not{v}}{2} Q_v(z) e^{i(q-m_Q v + m_{Q'}v')\cdot z} \int_k \int_{\tilde{k}} b_v^+(k,s) d_q^+(\tilde{k},\tilde{s}) \phi_{s\tilde{s}}(k,\tilde{k}) |0 > \\
&=< 0| \int_{k'} \int_{\tilde{k}'} \phi'^{\dagger}_{s'\tilde{s}'}(k',\tilde{k}') d_q(\tilde{k}',\tilde{s}') \delta(\bar{\Lambda}_{H'} v' - k' - \tilde{k}') \\
&(-iZ_2'^{-\frac{1}{2}}) \int_y \bar{u}(v',s') e^{ik'\cdot y} i\not{v}' v' \cdot \partial_y Q'_{v'}(y) \int_z \bar{Q}'_{v'}(z) \frac{1+\not{v}'}{2} \Gamma \frac{1+\not{v}}{2} Q_v(z) \\
&e^{i(q-m_Q v + m_{Q'}v')\cdot z} \int_k \int_{\tilde{k}} (-iZ_2^{-\frac{1}{2}}) \int_x \bar{Q}_v(x) (-i \overleftarrow{\partial}_x \cdot v \not{v}) u(v,s) e^{-ik\cdot x} \\
&d_q^+(\tilde{k},\tilde{s}) \phi_{s\tilde{s}}(k,\tilde{k}) \delta(\bar{\Lambda}_H v - k - \tilde{k}) |0 > \qquad (5.18)
\end{aligned}
$$

where the definition of meson state eq.(5.16) and the LSZ reduction formula for effective heavy quarks has been used in step 1 and step 2 respectively. Contracting

the heavy quark fields

$$Q'_{v'}(y)\bar{Q}'_{v'}(z) \to \int_{l'} e^{-il'\cdot(y-z)}\frac{i}{\not{v}'v'\cdot l'}$$
$$Q_v(z)\bar{Q}_v(x) \to \int_{l} e^{-il\cdot(z-x)}\frac{i}{\not{v}v\cdot l} \tag{5.19}$$

and integrating over x, y, z, l, l' as well as using the relation

$$< 0|d_q(\tilde{k}', \tilde{s}')d_q^+(\tilde{k}, \tilde{s})|0 >= \delta^3(\overrightarrow{\tilde{k}}' - \overrightarrow{\tilde{k}})\delta_{\tilde{s}'\tilde{s}}\frac{\tilde{k}^0}{m} \tag{5.20}$$

We obtain

$$\begin{aligned}
&T^0_\Gamma =< 0|\int_{k'}\int_{\tilde{k}'}\phi'^\dagger_{s'\tilde{s}'}(k', \tilde{k}')\delta(\bar{\Lambda}_{H'}v' - k' - \tilde{k}')(-iZ_2'^{-\frac{1}{2}}) \\
&\bar{u}(v', s')i\frac{1+\not{v}'}{2}\Gamma i\frac{1+\not{v}}{2}u(v, s)(-iZ_2^{-\frac{1}{2}})\int_k\int_{\tilde{k}}\delta(q - m_Q v - k + m_{Q'}v' + k') \\
&\delta(\tilde{k}' - \tilde{k})\phi_{s\tilde{s}}(k, \tilde{k})\delta(\bar{\Lambda}_H v - k - \tilde{k})|0 > \\
&= \frac{\delta(q - p + p')}{\sqrt{Z_2'Z_2}}\bar{u}(v', s')\frac{1+\not{v}'}{2}\Gamma\frac{1+\not{v}}{2}u(v, s)\int_{\tilde{k}}\phi'^\dagger_{s'\tilde{s}}(v'\cdot\tilde{k}, \tilde{k}^2)\phi_{s\tilde{s}}(v\cdot\tilde{k}, \tilde{k}^2) \\
&\equiv \frac{\delta(q - p + p')}{\sqrt{Z_2'Z_2}}\bar{u}(v', s')\frac{1+\not{v}'}{2}\Gamma\frac{1+\not{v}}{2}u(v, s)\int_{k}\phi'^\dagger_{s'\tilde{s}}(v'\cdot k, k^2)\phi_{s\tilde{s}}(v\cdot k, k^2)
\end{aligned} \tag{5.21}$$

In general, the above wave functions can be written into the following forms

$$\phi_{s\tilde{s}}(v\cdot\tilde{k}, \tilde{k}^2) = \bar{u}(v, s)i\gamma_5 v(\tilde{k}, \tilde{s})\varphi_P(v\cdot\tilde{k}, \tilde{k}^2) \tag{5.22}$$
$$\phi'_{s'\tilde{s}}(v'\cdot\tilde{k}, \tilde{k}^2) = \bar{v}(\tilde{k}, \tilde{s})i\gamma_5 u(v', s')\varphi'_P(v'\cdot\tilde{k}, \tilde{k}^2) \tag{5.23}$$

for pseudoscalar mesons, and

$$\phi_{s\tilde{s}}(v\cdot\tilde{k}, \tilde{k}^2) = \bar{u}(v, s)\not{\epsilon}v(\tilde{k}, \tilde{s})\varphi_V(v\cdot\tilde{k}, \tilde{k}^2) \tag{5.24}$$
$$\phi'_{s'\tilde{s}}(v'\cdot\tilde{k}, \tilde{k}^2) = \bar{v}(\tilde{k}, \tilde{s})\not{\epsilon}u(v', s')\varphi'_V(v'\cdot\tilde{k}, \tilde{k}^2) \tag{5.25}$$

for vector mesons. The spin-flavor symmetry implies that $\varphi_V = \varphi_P = \varphi$.

With the above formulation, the transition matrix can be simplified to be

$$\begin{aligned}
T^0_\Gamma &= < H'_{v'}|\int_z\bar{Q}'_{v'}(z)\frac{1+\not{v}'}{2}\Gamma\frac{1+\not{v}}{2}Q_v(z)e^{i(q-m_Qv+m_{Q'}v')\cdot z}|H_v > \\
&= \frac{\delta(q-p+p')}{\sqrt{Z_2'Z_2}}Tr\left(\bar{\mathcal{M}}_+(v')\Gamma\mathcal{M}_+(v)\frac{1}{\bar{\Lambda}}\int_{\tilde{k}}(1 - \frac{\not{\tilde{k}}}{m_q})\varphi'(v'\cdot\tilde{k}, \tilde{k}^2)\varphi(v\cdot\tilde{k}, \tilde{k}^2)\right) \\
&= \delta(q-p+p')\xi(v\cdot v')\ Tr\left(\bar{\mathcal{M}}_+(v')\Gamma\mathcal{M}_+(v)\right)
\end{aligned} \tag{5.26}$$

where the $\mathcal{M}_+(v)$ is the spin wave functions

$$\mathcal{M}_+(v) = \sqrt{\bar{\Lambda}} P_+ \begin{cases} -\gamma^5, & pseudoscalar\ meson\ \ P \\ \not{\epsilon}, & vector\ meson\ \ V \end{cases} \tag{5.27}$$

Here ϵ^μ is the polarization vector of the vector meson. In obtaining the above results, we have used the following property

$$\frac{1}{\bar{\Lambda}} \int_{\tilde{k}} (1 - \frac{\tilde{\not{k}}}{m_q}) \varphi'(v' \cdot \tilde{k}, \tilde{k}^2) \varphi(v \cdot \tilde{k}, \tilde{k}^2) = \zeta(v \cdot v')(1 - \alpha \not{v} - \alpha' \not{v}') \tag{5.28}$$

and introduced the function $\xi(v \cdot v')$ via the following definition

$$\xi(v \cdot v') = \frac{1}{\sqrt{Z_2' Z_2}} \zeta(v \cdot v')(1 + \alpha + \alpha') \tag{5.29}$$

which is the well-known universal Isgur-Wise function[15].

For the anti-meson state $H = \bar{M} = (\bar{q}Q)$, we have in general

$$|H_v >= \int_k \int_{\tilde{k}} \delta(\bar{\Lambda}_H v - k - \tilde{k}) d_v^+(k, s) b_q^+(\tilde{k}, \tilde{s}) \phi_{s\tilde{s}}(k, \tilde{k}) |0> \tag{5.30}$$

where $\phi_{s\tilde{s}}(k, \tilde{k})$ is the wave function in momentum space. In this case, only the second term in the transition matrix element T_Γ^0 becomes non-vanishing. Following the same procedure, one arrives at

$$\begin{aligned} T_\Gamma^0 &= < H'_{v'} | \int_z \bar{Q}'_{v'}(z) \frac{1 - \not{v}'}{2} \Gamma \frac{1 - \not{v}}{2} Q_v(z) e^{i(q - m_Q v + m_{Q'} v') \cdot z} | H_v > \\ &= \frac{\delta(q - p + p')}{\sqrt{Z_2' Z_2}} Tr \left(\bar{\mathcal{M}}_-(v') \Gamma \mathcal{M}_-(v) \frac{1}{\bar{\Lambda}} \int_{\tilde{k}} (1 + \frac{\tilde{\not{k}}}{m_q}) \varphi'(v' \cdot \tilde{k}, \tilde{k}^2) \varphi(v \cdot \tilde{k}, \tilde{k}^2) \right) \\ &= \delta(q - p + p') \xi(v \cdot v')\ Tr \left(\bar{\mathcal{M}}_-(v') \Gamma \mathcal{M}_-(v) \right) \end{aligned} \tag{5.31}$$

where the $\mathcal{M}_-(v)$ is the spin wave function for anti-meson state

$$\mathcal{M}_-(v) = \sqrt{\bar{\Lambda}} P_- \begin{cases} -\gamma^5, & pseudoscalar\ meson\ \ P \\ \not{\epsilon}, & vector\ meson\ \ V \end{cases} \tag{5.32}$$

In general, we have

$$\begin{aligned} T_\Gamma^0 &= < H'_{v'} | \bar{Q}'_{v'} \Gamma Q_v | H_v > \\ &= \xi(v \cdot v')\ Tr \left(\bar{\mathcal{M}}'(v') \Gamma \mathcal{M}(v) \right) \end{aligned} \tag{5.33}$$

Where $\bar{\mathcal{M}} = \gamma^0 \mathcal{M}^\dagger \gamma^0$ is the spin-wave function in HQEFT. It is specified to $\mathcal{M} = \mathcal{M}_+$ for heavy mesons and $\mathcal{M} = \mathcal{M}_-$ for heavy anti-mesons. $\xi(\omega)$ is the Isgur-Wise function that normalizes to unity at the point of zero recoil $\omega = 1$, i.e., $\xi(1) = 1$.

So far, we explicitly demonstrate the trace formula for the transition matrix elements in HQEFT at leading order. Generalization to higher orders is straightforward.

D. Transition Form Factors and $1/m_Q$ Expansions in HQEFT

We shall apply the trace formula obtained in the previous subsection to evaluate the hadronic matrix elements. In general, the hadronic matrix elements of vector and axial vector currents between pseudoscalars and vector mesons are described by 18 transition form factors

$$
\begin{aligned}
&< D(v')|\bar{c}\gamma^\mu b|B(v) >= \sqrt{m_D m_B}[h_+(\omega)(v+v')^\mu + h_-(\omega)(v-v')^\mu],\\
&< D^*(v',\epsilon')|\bar{c}\gamma^\mu b|B(v) >= i\sqrt{m_{D^*} m_B} h_V(\omega)\epsilon^{\mu\nu\alpha\beta}\epsilon'^*_\nu v'_\alpha v_\beta,\\
&< D^*(v',\epsilon')|\bar{c}\gamma^\mu\gamma^5 b|B(v) >= \sqrt{m_{D^*} m_B}[h_{A_1}(\omega)(1+\omega)\epsilon'^{*\mu} - h_{A_2}(\omega)(\epsilon'^*\cdot v)v^\mu\\
&\qquad -h_{A_3}(\omega)(\epsilon'^*\cdot v)v'^\mu],\\
&< D^*(v',\epsilon')|\bar{c}\gamma^\mu b|B^*(v,\epsilon) >= \sqrt{m_{D^*} m_{B^*}}\{-(\epsilon\cdot\epsilon'^*)[h_1(\omega)(v+v')^\mu + h_2(\omega)(v-v')^\mu]\\
&\qquad +h_3(\omega)(\epsilon'^*\cdot v)\epsilon^\mu + h_4(\omega)(\epsilon\cdot v')\epsilon'^{*\mu} - (\epsilon\cdot v')(\epsilon'^*\cdot v)[h_5(\omega)v^\mu + h_6(\omega)v'^\mu]\},\\
&< D^*(v',\epsilon')|\bar{c}\gamma^\mu\gamma^5 b|B^*(v,\epsilon) >= i\sqrt{m_{D^*} m_{B^*}}\{\epsilon^{\mu\nu\alpha\beta}\{\epsilon_\alpha\epsilon'^*_\beta[h_7(\omega)(v+v')_\nu\\
&\qquad +h_8(\omega)(v-v')_\nu] + v'_\alpha v_\beta[h_9(\omega)(\epsilon'^*\cdot v)\epsilon_\nu + h_{10}(\omega)(\epsilon\cdot v')\epsilon'^*_\nu]\}\\
&\qquad +\epsilon^{\alpha\beta\gamma\delta}\epsilon_\alpha\epsilon'^*_\beta v_\gamma v'_\delta[h_{11}(\omega)v^\mu + h_{12}(\omega)v'^\mu]\}. \qquad (5.34)
\end{aligned}
$$

To relate the form factors $h_i(\omega)$ with the matrix elements of operators in HQEFT, we reexpress the general form of transition matrix elements in HQEFT via the $1/m_Q$ expansion (5.6) to the following explicit forms

$$
\begin{aligned}
\mathcal{A} \; = \; &< H'_{v'}|\bar{Q}^{(+)}_{v'}\Gamma Q^{(+)}_v|H_v >\\
&+\frac{1}{2m_Q} < H'_{v'}|\bar{Q}^{(+)}_{v'}\Gamma\frac{1}{iv\cdot\partial}P_+[D^2_\perp + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}]Q^{(+)}_v|H_v >\\
&-\frac{1}{2m_{Q'}} < H'_{v'}|\bar{Q}^{(+)}_{v'}[\overleftarrow{D}^2_\perp + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}]P'_+\frac{1}{iv'\cdot\overleftarrow{\partial}}\Gamma Q^{(+)}_v|H_v >
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{4m_Q^2} < H'_{v'}|\bar{Q}_{v'}^{(+)}\Gamma\frac{1}{iv\cdot\partial}P_+[iD_\perp^2(v\cdot D)-iD^\alpha v^\beta F_{\alpha\beta}+D_\perp^2\frac{1}{iv\cdot\partial}P_+D_\perp^2\\
&-\frac{1}{2}\sigma_{\alpha\beta}F^{\alpha\beta}(v\cdot D)-\sigma^{\sigma\alpha}v^\beta D_\sigma F_{\alpha\beta}+D_\perp^2\frac{1}{iv\cdot\partial}P_+\frac{i}{2}\sigma_{\gamma\sigma}F^{\gamma\sigma}\\
&+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\frac{1}{iv\cdot\partial}P_+D_\perp^2-\frac{1}{4}\sigma_{\alpha\beta}F^{\alpha\beta}\frac{1}{iv\cdot\partial}P_+\sigma_{\gamma\sigma}F^{\gamma\sigma}]Q_v^{(+)}|H_v>\\
&-\frac{1}{4m_{Q'}^2} < H'_{v'}|\bar{Q}_{v'}^{(+)}[i(v'\cdot\overleftarrow{D})\overleftarrow{D}_\perp^2-iF_{\alpha\beta}\overleftarrow{D}^\alpha v'^\beta+\overleftarrow{D}_\perp^2 P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\overleftarrow{D}_\perp^2\\
&-\frac{1}{2}(v'\cdot\overleftarrow{D})\sigma_{\alpha\beta}F^{\alpha\beta}-F_{\alpha\beta}\overleftarrow{D}_\sigma v'^\beta\sigma^{\sigma\alpha}+\frac{i}{2}\sigma_{\gamma\sigma}F^{\gamma\sigma}P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\overleftarrow{D}_\perp^2\\
&+\overleftarrow{D}_\perp^2 P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}-\frac{1}{4}\sigma_{\gamma\sigma}F^{\gamma\sigma}P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\sigma_{\alpha\beta}F^{\alpha\beta}]P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\Gamma Q_v^{(+)}|H_v>\\
&+\frac{1}{4m_{Q'}m_Q} < H'_{v'}|\bar{Q}_{v'}^{(+)}[\overleftarrow{D}_\perp^2 P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\Gamma\frac{1}{iv\cdot\partial}P_+D_\perp^2\\
&+\overleftarrow{D}_\perp^2 P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\Gamma\frac{1}{iv\cdot\partial}P_+\frac{i}{2}\sigma_{\gamma\sigma}F^{\gamma\sigma}+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\Gamma\frac{1}{iv\cdot\partial}P_+D_\perp^2\\
&-\frac{1}{4}\sigma_{\alpha\beta}F^{\alpha\beta}P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\Gamma\frac{1}{v\cdot\partial}P_+\sigma_{\gamma\sigma}F^{\gamma\sigma}]Q_v^{(+)}|H_v> \qquad (5.35)
\end{aligned}
$$

with $\sigma^{\alpha\beta}=\frac{i}{2}[\gamma^\alpha,\gamma^\beta]$, $P_+=\frac{1+\not{v}}{2}$ and $P'_+=\frac{1+\not{v}'}{2}$ being project operators. Where $F^{\alpha\beta}=[D^\beta,D^\alpha]$ is the field strength of gluons tensor.

One can now adopt the trace formulation approach to parameterize the relevant matrix elements via Lorentz invariance

$$
\begin{aligned}
&< M'_{v'}|\bar{Q}_{v'}^{(+)}\Gamma Q_v^{(+)}|M_v>=-\xi(\omega)Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}],\\
&< M'_{v'}|\bar{Q}_{v'}^{(+)}\Gamma\frac{1}{iv\cdot\partial}P_+D_\perp^2Q_v^{(+)}|M_v>=\kappa_1(\omega)\frac{1}{\bar{\Lambda}}Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}],\\
&< M'_{v'}|\bar{Q}_{v'}^{(+)}\Gamma\frac{1}{iv\cdot\partial}P_+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}Q_v^{(+)}|M_v>=-\frac{1}{\bar{\Lambda}}Tr[\kappa_{\alpha\beta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}\mathcal{M}],\\
&< M'_{v'}|\bar{Q}_{v'}^{(+)}\Gamma\frac{1}{iv\cdot\partial}P_+[iD_\perp^2(v\cdot D)-iD^\alpha v^\beta F_{\alpha\beta}]Q_v^{(+)}|M_v>=-\varrho_1(\omega)\frac{1}{\bar{\Lambda}}Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}],\\
&< M'_{v'}|\bar{Q}_{v'}^{(+)}\Gamma\frac{1}{iv\cdot\partial}P_+[\frac{1}{2}\sigma_{\alpha\beta}F^{\alpha\beta}(v\cdot D)+\sigma^{\sigma\alpha}v^\beta D_\sigma F_{\alpha\beta}]Q_v^{(+)}|M_v>\\
&=-\frac{1}{\bar{\Lambda}}Tr[\varrho_{\alpha\beta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}\mathcal{M}],\\
&< M'_{v'}|\bar{Q}_{v'}^{(+)}\Gamma\frac{1}{iv\cdot\partial}P_+D_\perp^2\frac{1}{iv\cdot\partial}P_+D_\perp^2Q_v^{(+)}|M_v>=-\chi_1(\omega)\frac{1}{\bar{\Lambda}^2}Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}],\\
&< M'_{v'}|\bar{Q}_{v'}^{(+)}\Gamma\frac{1}{iv\cdot\partial}P_+[D_\perp^2\frac{1}{iv\cdot\partial}P_+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\frac{1}{iv\cdot\partial}P_+D_\perp^2]Q_v^{(+)}|M_v>\\
&=\frac{1}{\bar{\Lambda}^2}Tr[\chi_{\alpha\beta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}\mathcal{M}],
\end{aligned}
$$

$$
\begin{aligned}
&< M'_{v'}|\bar{Q}^{(+)}_{v'}\Gamma\frac{1}{iv\cdot\partial}P_+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\frac{1}{iv\cdot\partial}P_+\frac{i}{2}\sigma_{\gamma\sigma}F^{\gamma\sigma}Q^{(+)}_v|M_v>\\
&=-\frac{1}{\bar{\Lambda}^2}Tr[\chi_{\alpha\beta\gamma\sigma}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}P_+\frac{i}{2}\sigma^{\gamma\sigma}\mathcal{M}],\\
&< M'_{v'}|\bar{Q}^{(+)}_{v'}\overleftarrow{D}^2_\perp P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\Gamma\frac{1}{iv\cdot\partial}P_+D^2_\perp Q^{(+)}_v|M_v>=-\eta_1(\omega)\frac{1}{\bar{\Lambda}^2}Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}],\\
&< M'_{v'}|\bar{Q}^{(+)}_{v'}\overleftarrow{D}^2_\perp P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\Gamma\frac{1}{iv\cdot\partial}P_+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}Q^{(+)}_v|M_v>\\
&=\frac{1}{\bar{\Lambda}^2}Tr[\eta_{\alpha\beta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}\mathcal{M}],\\
&< M'_{v'}|\bar{Q}^{(+)}_{v'}\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}P'_+\frac{1}{-iv'\cdot\overleftarrow{\partial}}\Gamma\frac{1}{iv\cdot\partial}P_+\frac{i}{2}\sigma_{\gamma\sigma}F^{\gamma\sigma}Q^{(+)}_v|M_v>\\
&=-\frac{1}{\bar{\Lambda}^2}Tr[\eta_{\alpha\beta\gamma\sigma}(v,v')\bar{\mathcal{M}}'\frac{i}{2}\sigma^{\alpha\beta}P'_+\Gamma P_+\frac{i}{2}\sigma^{\gamma\sigma}\mathcal{M}]. \qquad (5.36)
\end{aligned}
$$

with $\omega = v\cdot v'$. Where $\bar{\mathcal{M}} = \gamma^0\mathcal{M}^\dagger\gamma^0$ is the spin-wave function in HQEFT (it is specified to $\mathcal{M} = \mathcal{M}_+$ for heavy mesons and $\mathcal{M} = \mathcal{M}_-$ for heavy anti-mesons). $\xi(\omega)$ is the Isgur-Wise function that normalizes to unity at the point of zero recoil $\omega = 1$, i.e., $\xi(1) = 1$. $\kappa_{\alpha\beta}(v,v')$, $\varrho_{\alpha\beta}(v,v')$, $\chi_{\alpha\beta}(v,v')$, $\eta_{\alpha\beta}(v,v')$, $\chi_{\alpha\beta\gamma\delta}(v,v')$, and $\eta_{\alpha\beta\gamma\delta}(v,v')$ are the Lorentz tensors. They can be decomposed into scalar form factors in terms of the Lorentz tensor $g_{\mu\nu}$ as well as the Lorentz vectors γ_μ, v_μ and v'_μ. Their Lorentz decompositions are found to have the following general forms

$$
\begin{aligned}
&\kappa_{\alpha\beta}(v,v') = i\kappa_2(\omega)\sigma_{\alpha\beta} + \kappa_3(\omega)(v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha),\\
&\varrho_{\alpha\beta}(v,v') = i\varrho_2(\omega)\sigma_{\alpha\beta} + \varrho_3(\omega)(v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha),\\
&\chi_{\alpha\beta}(v,v') = i\chi_2(\omega)\sigma_{\alpha\beta} + \chi_3(\omega)(v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha),\\
&\eta_{\alpha\beta}(v,v') = i\eta_2(\omega)\sigma_{\alpha\beta} + \eta_3(\omega)(v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha),\\
&\chi_{\alpha\beta\gamma\delta}(v,v') = \chi_4(\omega)(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) + \chi_5(\omega)\sigma_{\gamma\delta}\sigma_{\alpha\beta}\\
&+i\chi_6(\omega)(g_{\alpha\gamma}\sigma_{\beta\delta} - g_{\beta\gamma}\sigma_{\alpha\delta} - g_{\alpha\delta}\sigma_{\beta\gamma} + g_{\beta\delta}\sigma_{\alpha\gamma}) + \chi_7(\omega)(v'_\gamma\gamma_\delta - v'_\delta\gamma_\gamma)(v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha)\\
&+\chi_8(\omega)(g_{\alpha\gamma}v'_\beta v'_\delta - g_{\beta\gamma}v'_\alpha v'_\delta - g_{\alpha\delta}v'_\beta v'_\gamma + g_{\beta\delta}v'_\alpha v'_\gamma)\\
&+\chi_9(\omega)(g_{\alpha\gamma}v'_\beta\gamma_\delta - g_{\beta\gamma}v'_\alpha\gamma_\delta - g_{\alpha\delta}v'_\beta\gamma_\gamma + g_{\beta\delta}v'_\alpha\gamma_\gamma)\\
&+\chi_{10}(\omega)(g_{\alpha\gamma}\gamma_\beta v'_\delta - g_{\beta\gamma}\gamma_\alpha v'_\delta - g_{\alpha\delta}\gamma_\beta v'_\gamma + g_{\beta\delta}\gamma_\alpha v'_\gamma)\\
&+i\chi_{11}(\omega)\times(\sigma_{\alpha\gamma}v'_\beta\gamma_\delta - \sigma_{\beta\gamma}v'_\alpha\gamma_\delta - \sigma_{\alpha\delta}v'_\beta\gamma_\gamma + \sigma_{\beta\delta}v'_\alpha\gamma_\gamma)\\
&+i\chi_{12}(\omega)(\sigma_{\alpha\gamma}\gamma_\beta v'_\delta - \sigma_{\beta\gamma}\gamma_\alpha v'_\delta - \sigma_{\alpha\delta}\gamma_\beta v'_\gamma + \sigma_{\beta\delta}\gamma_\alpha v'_\gamma),
\end{aligned}
$$

$$
\begin{aligned}
&\eta_{\alpha\beta\gamma\delta}(v,v') = \eta_4(\omega)(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) + \eta_5(\omega)\sigma_{\gamma\delta}\sigma_{\alpha\beta} \\
&+i\eta_6(\omega)(g_{\alpha\gamma}\sigma_{\beta\delta} - g_{\beta\gamma}\sigma_{\alpha\delta} - g_{\alpha\delta}\sigma_{\beta\gamma} + g_{\beta\delta}\sigma_{\alpha\gamma}) + \eta_7(\omega)(v'_\gamma\gamma_\delta - v'_\delta\gamma_\gamma)(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha) \\
&+\eta_8(\omega)(g_{\alpha\gamma}v_\beta v'_\delta - g_{\beta\gamma}v_\alpha v'_\delta - g_{\alpha\delta}v_\beta v'_\gamma - g_{\beta\delta}v_\alpha v'_\gamma) \\
&+\eta_9(\omega)(g_{\alpha\gamma}v_\beta\gamma_\delta - g_{\beta\gamma}v_\alpha\gamma_\delta - g_{\alpha\delta}v_\beta\gamma_\gamma + g_{\beta\delta}v_\alpha\gamma_\gamma + g_{\alpha\gamma}\gamma_\beta v'_\delta - g_{\beta\gamma}\gamma_\alpha v'_\delta - g_{\alpha\delta}\gamma_\beta v'_\gamma \\
&+g_{\beta\delta}\gamma_\alpha v'_\gamma) + i\eta_{10}(\omega)(v_\beta\gamma_\delta\sigma_{\alpha\gamma} - v_\alpha\gamma_\delta\sigma_{\beta\gamma} - v_\beta\gamma_\gamma\sigma_{\alpha\delta} + v_\alpha\gamma_\gamma\sigma_{\beta\delta} + \sigma_{\alpha\gamma}\gamma_\beta v'_\delta \\
&-\sigma_{\beta\gamma}\gamma_\alpha v'_\delta - \sigma_{\alpha\delta}\gamma_\beta v'_\gamma + \sigma_{\beta\delta}\gamma_\alpha v'_\gamma). \qquad (5.37)
\end{aligned}
$$

where the identity $v_\alpha P_+ \sigma^{\alpha\beta}\mathcal{M} = 0$ has been used. Thus all matrix elements up to order of $1/m_Q^2$ can be represented by the above scalar form factors.

In general, a heavy quark within a hadron cannot be on-mass shell due to strong interactions among heavy quark and light quark as well as soft gluons. The off-mass shellness of the heavy quark in the heavy hadron is characterized by a residual momentum $k = \bar{\Lambda}v + \tilde{k}$. The total momentum p of the heavy quark in a hadron may be written as: $p = m_Q v + k = \hat{m}_Q v + \tilde{k}$. Thus the residual momentum $k = \bar{\Lambda}v + \tilde{k}$ of the heavy quark within a hadron is assumed to comprise the main contributions of the light degrees of freedom. Where $\tilde{k}$ is the part which depends on heavy flavor and is suppressed by $1/m_Q$. With this picture the heavy quark may be regarded as a 'dressed heavy quark', and the heavy hadron containing a single heavy quark is more reliable to be considered as a dualized particle of a 'dressed heavy quark'. For this reason, the form factors defined in HQEFT should have a very weak dependence on the light constituents of heavy hadrons. Thus it is useful in HQEFT to define the 'dressed heavy quark' mass as

$$
\hat{m}_Q \equiv \lim_{m_Q \to \infty} m_H = m_Q + \bar{\Lambda}. \qquad (5.38)
$$

As the momentum $\tilde{k} = p - \hat{m}_Q v$ carried by the effective heavy quark field Q_v within the heavy hadron is expected to be much smaller than the binding energy $\bar{\Lambda}$, so we can make the following expansion

$$
\frac{1}{iv\cdot\partial} \to \frac{1}{v\cdot k} \to \frac{1}{\bar{\Lambda} + v\cdot\tilde{k}} = \frac{1}{\bar{\Lambda}}\left(1 + O(\frac{v\cdot\tilde{k}}{\bar{\Lambda}})\right) \sim \frac{1}{\bar{\Lambda}}. \qquad (5.39)
$$

Here $\bar{\Lambda}$ characterizes the effects of the light degrees of freedom in the heavy hadron due to nonperturbative effects. Therefore, from the point of view in HQEFT, the

'dressed heavy quark'-hadron duality should become more reliable than the naive heavy quark-hadron duality.

After completing the trace calculation, one can easily write down the matrix elements of vector and axial vector currents between pseudoscalar and vector mesons in terms of Lorentz scalar factors κ_i ,ϱ_i, χ_i and η_i. The most general results at non-zero recoil are quite lengthy[4]. At zero recoil point, one only needs to know four necessary form factors which have the following simple forms

$$\begin{aligned}
h_+(1) &= 1 + \frac{1}{8\bar{\Lambda}^2}(\frac{1}{m_b} - \frac{1}{m_c})^2[\ (\kappa_1 + 3\kappa_2)^2 - (\eta_1 + 3\eta_2 - 3\eta_4 - 9\eta_5 - 6\eta_6)\ ], \\
h_{A_1}(1) &= 1 + \frac{1}{8m_b^2\bar{\Lambda}^2}\{[(\kappa_1 + 3\kappa_2) - \frac{m_b}{m_c}(\kappa_1 - \kappa_2)]^2 - (\eta_1 + 3\eta_2 - 3\eta_4 - 9\eta_5 - 6\eta_6)\} \\
&\quad - \frac{1}{8m_c^2\bar{\Lambda}^2}(\eta_1 - \eta_2 - 3\eta_4 - \eta_5 + 2\eta_6) + \frac{1}{4m_b m_c\bar{\Lambda}^2}(\eta_1 + \eta_2 + \eta_4 + 3\eta_5 + 2\eta_6), \\
h_1(1) &= 1 + \frac{1}{8\bar{\Lambda}^2}(\frac{1}{m_b} - \frac{1}{m_c})^2[\ (\kappa_1 - \kappa_2)^2 - (\eta_1 - \eta_2 - 3\eta_4 - \eta_5 + 2\eta_6)\ ], \\
h_7(1) &= -1 - \frac{1}{8\bar{\Lambda}^2}(\frac{1}{m_b} - \frac{1}{m_c})^2(\kappa_1 - \kappa_2)^2 + \frac{1}{8\bar{\Lambda}^2}(\frac{1}{m_b^2} + \frac{1}{m_c^2})(\eta_1 - 2\eta_2 - 3\eta_4 \\
&\quad - \eta_5 + 2\eta_6) - \frac{1}{4m_b m_c\bar{\Lambda}^2}(\eta_1 - 2\eta_2 + \eta_4 - \eta_5 - 2\eta_6)
\end{aligned} \tag{5.40}$$

where the normalization of $\xi(\omega)$ at zero recoil $\xi(\omega = 1) = 1$ has been used. The meson masses are found from the normalization condition to be

$$\begin{aligned}
m_{D(B)} &= m_{c(b)} + \bar{\Lambda}_{D(B)} = \hat{m}_{c(b)} - (\frac{1}{m_{c(b)}} - \frac{\bar{\Lambda}}{2m_{c(b)}^2})(\kappa_1 + 3\kappa_2) \\
&- \frac{1}{2m_{c(b)}^2\bar{\Lambda}}(\varrho_1\bar{\Lambda} + 3\varrho_2\bar{\Lambda} + \chi_1 + 3\chi_2 - 3\chi_4 - 9\chi_5 - 6\chi_6) \\
&+ \frac{1}{4m_{c(b)}^2\bar{\Lambda}}(\eta_1 + 6\eta_2 - 3\eta_4 - 9\eta_5 - 6\eta_6) + O(\frac{1}{m_{c(b)}^3}),
\end{aligned} \tag{5.41}$$

$$\begin{aligned}
m_{D^*(B^*)} &= m_{c(b)} + \bar{\Lambda}_{D^*(B^*)} = \hat{m}_{c(b)} - (\frac{1}{m_{c(b)}} - \frac{\bar{\Lambda}}{2m_{c(b)}^2})(\kappa_1 - \kappa_2) \\
&- \frac{1}{2m_{c(b)}^2\bar{\Lambda}}(\varrho_1\bar{\Lambda} - \varrho_2\bar{\Lambda} + \chi_1 - \chi_2 - 3\chi_4 - \chi_5 + 2\chi_6) \\
&+ \frac{1}{4m_{c(b)}^2\bar{\Lambda}}(\eta_1 - 2\eta_2 - 3\eta_4 - \eta_5 + 2\eta_6) + O(\frac{1}{m_{c(b)}^3})
\end{aligned} \tag{5.42}$$

It is of interest to note that two form factors $h_-(\omega)$ and $h_2(\omega)$ in HQEFT vanish in the whole region of momentum transfer, i.e., $h_-(\omega) = h_2(\omega) = 0$. Such a feature

results from the fact that in HQEFT the operators in the effective Lagrangian and effective current contain only terms with even powers of $\not{D}_\perp$ due to the contributions of quark-antiquark interaction terms. Generally, the mesonic matrix elements up to the second order power corrections can be described by a set of 29 scalar form factors, which are universal functions of the kinematic variable $\omega = v \cdot v'$. Such a number is less than the one introduced in the widely used heavy quark effective theory containing no quark-antiquark coupling terms, where 34 form factors are needed. All the interesting features are attributed to the quark-antiquark interaction terms in HQEFT. At zero recoil point, some of the form factors are kinematically suppressed, only 15 universal form factors are needed to describe the mesonic matrix elements up to order $1/m_Q^2$. Where κ_1 and κ_2 characterize the contributions of the order $1/m_Q$ operators at zero recoil. As seen from the above results, the first order corrections to the meson mass arise only from those two form factors. They play the same roles as the parameters λ_1 and λ_2 defined in the heavy quark effective theory without quark-antiquark coupling terms. It is seen that the order $1/m_Q$ corrections in all meson transition matrix elements of weak currents are automatically absent at zero recoil in HQEFT after including the contributions from the quark-antiquark interaction terms. The absence of $1/m_Q$ order corrections at zero recoil was first noticed in ref.[23] for the matrix element in the $B \to D^*$ transition, a detailed proof was made by adopting equation of motion based on the widely used heavy quark effective theory containing no quark-antiquark coupling terms. Nevertheless, in that treatment unlike to HQEFT, some transition processes like $B \to D$ transition remain receiving $1/m_Q$ corrections, which actually displays an explicit difference between two effective theories.

E. Interesting Features in Applications of HQEFT

It is seen from above analyzes that the theoretical framework of HQEFT provides a powerful tool for systematically evaluating the hadronic matrix elements via $1/m_Q$ expansion, it allows us to explore its applications to heavy quark systems. Here we only outline the most interesting features observed in the applications of HQEFT.

It of interest to note that the quark-antiquark interacting terms in HQEFT play an important role for understanding the heavy hadron dynamics. Firstly, only the even powers of $\not{D}_\perp$ appear in the effective Hamiltonian for either quark fields or anti-quark fields when including the contributions from the quark-antiquark interacting terms, which significantly simplifies the structure of transition matrix elements in the $1/m_Q$ expansion of HQEFT. For instance, the $1/m_Q$ corrections from current expansion and from insertion of $1/m_Q$ order Lagrangian are attributed to the same set of wave functions $\kappa_i(\omega)$ $(i = 1, 2, 3)$, and the $1/m_Q$ order corrections to meson masses are related to the zero recoil values of wave functions $\kappa_1 \equiv \kappa_1(\omega = 1)$ and $\kappa_2 \equiv \kappa_2(\omega = 1)$. Thus at $1/m_Q$ order in HQEFT only 3 independent functions are involved in both the weak transition matrix elements and ground state meson masses. This feature allows us to determine certain wave functions from the meson masses.

Secondly, it has been shown in HQEFT that the order $1/m_Q$ corrections in weak transition matrix elements between all ground state mesons are automatically absent at zero recoil when including the contributions from the quark-antiquark coupling terms. Such a feature allows us to extract the CKM matrix element $|V_{cb}|$ not only from the exclusive semileptonic decay modes $B \to D^* l\nu$, but also from $B \to Dl\nu$ decay modes. More precise extraction of $|V_{cb}|$ and $|V_{ub}|$ up to the $1/m_Q^2$ order corrections have been carried out [4, 5, 6] within the framework of HQEFT.

Thirdly, as HQEFT can describe a slightly off-mass shell heavy quark within a hadron, it is believed that HQEFT should also lead to a consistent application for inclusive heavy hadron processes. Introducing the concept of 'dressed heavy quark' -hadron duality based on HQEFT becomes more reasonable than a naive quark-hadron duality. Here the "dressed heavy quark" mass is defined as $\hat{m}_Q = m_Q + \bar{\Lambda}$ which is related to the hadron mass at a high order $1/m_Q$ corrections, explicitly, we have $\hat{m}_Q = m_H[1 + O(1/m_Q^2)]$. Thus the mass quantity entering into the inclusive decay rates in the HQEFT formulation is the well defined "dressed heavy quark" mass $\hat{m}_Q = m_Q + \bar{\Lambda}$ rather than the heavy quark mass m_Q. As a consequence, the resulting inclusive decay rate formulae of heavy hadrons are found to receive no $1/\hat{m}_Q$ order corrections when the decay rates are expressed in terms of the physical heavy

hadron masses. This is the most interesting advantages of HQEFT, which allows us to perform the heavy quark expansion at the point of the well-defined "dressed heavy quark" mass $\hat{m}_Q$ instead of the heavy quark mass m_Q. Such a treatment successfully suppresses the next to leading order contributions of the expansion and diminishes the possible large uncertainties from the heavy quark mass m_Q. Of particular, we are naturally led to a consistent explanation for the long term puzzle of life time differences among bottom hadrons, i,e., the resulting predictions for the ratios $\tau(B_s^0)/\tau(B^0)$ and $\tau(\Lambda_b)/\tau(B^0)$[7] are remarkably consistent with the experimental data. In fact, only in this sense, we can simultaneously present a more precise and consistent determination for $|V_{cb}|$ and $|V_{ub}|$ from the inclusive bottom hadron decays. The numerical results for $|V_{cb}|$ and $|V_{ub}|$ were presented in ref.[7] up to the order of $1/m_Q^2$ corrections as $1/m_Q$ order corrections are automatically absent in HQEFT.

Finally, we would like to address that as $1/m_Q$ corrections can systematically be computed and consistently be estimated by the powers of $\bar{\Lambda}/m_Q$ in HQEFT. The leading behavior in the heavy quark expansion of HQEFT can characterize the main features for heavy quark systems. For instance, the scaling law for the heavy meson decay constants was truly found in HQEFT to hold in a good approximation for heavy bottom mesons[8].

VI. CONCLUSIONS AND REMARKS

We have shown that a large component QCD (LCQCD) with both large component effective heavy quark and antiquark fields can directly be derived from full QCD by integrating over the small components of heavy quark and antiquark fields with $|\mathbf{p}| < E+m_Q$. When the heavy quark is slightly off-mass shell with the residual momentum $k = p - m_Q v$ satisfying, at the rest frame $v = (1,0,0,0)$, the condition $|\mathbf{k}| \ll \sqrt{2k^0 m_Q} \sim \sqrt{2\bar{\Lambda} m_Q}$ for $k^0 \sim \bar{\Lambda}$, the typical case is for the heavy-light hadron system with $k^0 \sim |\mathbf{k}| \sim \bar{\Lambda} \ll m_Q$, then LCQCD can well be treated as a heavy quark effective field theory (HQEFT) via a systematical heavy quark expansion in terms of $1/m_Q$ once the contributions from the effective heavy quark-antiquark coupling terms are considered. Its leading term characterizes the behavior of heavy quarks in

the infinite mass limit and explicitly displays the heavy quark spin-flavor symmetry.

It has been seen that a basic theoretical framework of HQEFT can be established via an alternative quantization procedure instead of the usual canonical quantization, which includes the quantum generators of Poincare group, the Hilbert and Fock space, anticommutations and velocity superselection rule, propagator and Feynman rules, finite mass corrections and renormalization in HQEFT. The Lorentz invariance and discrete symmetries in HQEFT have explicitly been checked. In addition to spin-flavor symmetry, we have demonstrated some new symmetries in the infinite mass limit. The trivialization of gluon couplings and decouple theorem as well as the renormalization of Wilson loop have also been discussed.

The weak transition matrix elements have been well defined in HQEFT with a manifest spin-flavor symmetry in the infinite mass limit. From the vector current conservation and the well defined normalization of hadronic matrix element of vector current in the full QCD theory and HQEFT, the heavy hadron mass can alternatively be defined by the heavy quark mass and binding energy via $1/m_Q$ expansion, so that the heavy hadron masses are related to the transition wave functions at zero recoil and can be used to determined the numerical values of transition wave functions at zero recoil. In particular, we have demonstrated that a simple trace formulation for evaluating the transition matrix elements in HQEFT is derivable by using the LSZ reduction formula. It has explicitly been shown that the universal Isgur-Wise function of transition matrix elements is related to the overlapping integral between the wave functions of initial and final meson states. It is of interest to note that the trace formulation approach is very powerful for parameterizing the transition form factors via $1/m_Q$ expansion in HQEFT.

In summary, a complete theoretical framework of HQEFT has been established from QCD in the sense of effective quantum field theory, where the effective heavy quark and antiquark fields have been dealt with on the same footing in a fully symmetric way. The large component effective heavy quark-antiquark coupling terms have been shown to play an important role for analyzing $1/m_Q$ corrections when the "longitudinal" and "transverse" residual momenta of heavy quark are at the same order of power counting in the $1/m_Q$ expansion. It should be not surprised to

understand the results presented in [4, 5, 6, 7, 8, 9] for their consistency with the experimental data and theoretical expectations. We believe that more precise and consistent results can be made from HQEFT which will further be tested by more accurate experimental data based on B-factories and colliders.

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